COMPUTATIONAL ASTRONOMY AND THE EARLIEST VISIBILITY OF LUNAR CRESCENT

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ABSTRACT

Basic techniques of Computational Astronomy are reviewed and presented as the essential tools for simulation of Lunar phenomena. The importance of accurate determination of Julian Date and the Local Sidereal Time is discussed that are essential to determine the local time of sunset and the local coordinates of any object at that time. During the 20th century, a number of authors have contributed towards the understanding of the problem of earliest sighting of crescent Moon. The work of Maunder, Schoch, Bruin and Schaefer has been crucial in the development of this understanding. More recently, the work of Yallop, Ilyas, Ahmed and Shaukat has received great recognition. The work of Ahmed and Shaukat has been based mostly on the Yallop's Criterion. However almost all the modesl are based on the observational data of Schmidt who made observations from Athens for over 20 years during the late 19th century. In this work, a model of q-values developed by Yallop is analyzed in view of Maunders and the Indian Criteria along with the actual semi-diameter of the crescent Moon. The basic criterion is modified on the basis of data more recently collected.

1. INTRODUCTION

Observations or results of experiments form the basis of all theories describing any physical phenomenon. More minute observations and sophisticated experiments provide testing grounds for these theories. Kepler[18][26] deduced his laws, describing motion of planets, based on data available in his time. On the basis of his dynamical theories Newton proved Kepler's laws. However, Kepler's Laws of Planetary motion are valid only in view of the Two-Body problem[4][26]. For instance, the orbits of planets that are assumed elliptical by Kepler do not remain elliptical even if a Three-Body problem is considered. Much later, the Newtonian Dynamics that may be effectively used to describe n-Body problem, itself failed to describe the variations in the orbit of Mercury. Even the Relativistic Mechanics of Einstein could not completely account for this variation. Description of these Physical theories and putting them to work to determine the position of celestial bodies on our skies are two different aspect of the same effort. Observed positions lead to theories and a theory has to be tested by observations.

In the first part of this paper, tools of Computational Astronomy are presented, in view of the problem of determining the position of a planet or a satellite in its orbit and as it appears on the observable sky. In particular, the Two-Body[26] and the Three-Body[4] Problems are briefly reviewed and how they are used to determine positions of the Sun and the Moon in the sky. In the second part the problem of earliest visibility/sighting of New Crescent Moon is presented. The work of a number of contributors and authors is reviewed. Yallop's comparison[29] of the Maunder's[13], the Indian[11] and the Bruin's[3] methods is extended to include a q-test based on third degree polynomial fitted to data using Least Square Approximation. Moreover the possibility of better coefficients in the 2nd degree curve is considered to be closer to real observations.

In the end, astronomical conditions for a number of recently recorded observations calculated on the basis of each method are presented with a critical discussion.

2. COMPUTATIONAL ASTRONOMY

In this work, all the tools of Computational Astronomy are based on Newtonian Mechanics. The time argument used is based on Julian Date according to Gregorian Calendar[17]. The Julian Date is thus the time elapsed since the Noon at Greenwich on Monday, November 24 of the year -4713. It is intended that this work shall be extended to higher degree of precision in order to develop an indigenous and independent simulation for the celestial phenomena considered in this work. Most of the tools discussed in this article and the next one are due to Smart[26], Danby[4] and notes from the Astronomical Almanac[28].

2.1 *SPHERICAL TRIGONOMETRY*

We observe all celestial objects moving on the "Celestial Sphere". To determine the relative position of objects, angular separation between them and direction of one from the other we require Spherical Trigonometry, the study of triangle on a sphere. The shortest "distance" between two points on a sphere is along a "great circle arc". A "great circle" is the intersection of a plane passing through the centre of the sphere with the sphere. Any plane that does not pass through the center of the sphere intersects with the sphere in a "small circle".

Let A, B and C be points on a sphere. AB, BC and AC are great circle arcs with "length" equal to c, a and b respectively, given in angular measures as shown in Figure 1 page 3. Thus $a = \angle COB, b = \angle AOC$ and $c = \angle AOB$, where O is the centre of the sphere. The basic formulas in spherical trigonometry are:

$$\boldsymbol{Cos}(\boldsymbol{a}) = \boldsymbol{Cos}(\boldsymbol{b})\boldsymbol{Cos}(\boldsymbol{c}) + \boldsymbol{Sin}(\boldsymbol{b})\boldsymbol{Sin}(\boldsymbol{c})\boldsymbol{Cos}(\boldsymbol{A}) \quad (2.1)$$

$$\frac{\boldsymbol{Sin}(\boldsymbol{A})}{\boldsymbol{Sin}(\boldsymbol{a})} = \frac{\boldsymbol{Sin}(\boldsymbol{B})}{\boldsymbol{Sin}(\boldsymbol{b})} = \frac{\boldsymbol{Sin}(\boldsymbol{C})}{\boldsymbol{Sin}(\boldsymbol{c})} \quad (2.2)$$

Where ***A*** is the spherical angle between the sides ***b*** and ***c*** which is the angle between the planes the circular sectors AOC and AOB. Similarly the spherical angles ***B*** and ***C*** are defined. All the other formulas of spherical trigonometry can be obtained using these two formulas. First of the above two formulas, (2.1), gives the angular separation '***a***' between points ***B*** and ***C*** on the sphere. Direction of a point say ***A*** from a point ***B*** is given by the spherical angle ***C***. The two formulas can also be used to find the "shortest distances" between two places on Earth and the direction of one place from another, for instance direction of Qibla from any place on the Earth.

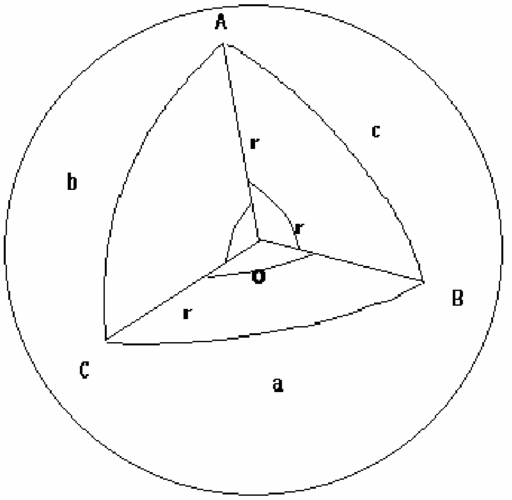

Figure 1

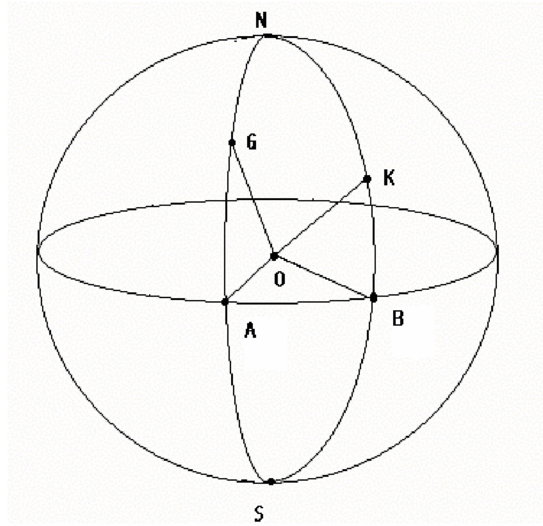

Figure 2

2.2 ***TERRESTRIAL COORDINATE SYSTEMS***

Latitudes and ***Longitudes*** give position of any point on the surface of earth. As shown in Figure 2 above, the reference for Latitude is the ***Equator***, the great circle that passes through the centre of Earth and its plane is perpendicular to the axis of rotation of Earth. Semi great circles joining the North Pole(point of intersection of the surface of Earth with the axis of rotation in the northern hemisphere) and the South Pole are called ***Meridians***. The Meridian Passing through Greenwich, near London, England is called the ***standard meridian***. In figure 2, G is Greenwich and K is any other place, say Karachi. NGAS is the standard meridian and NKBS the local meridian of Karachi. If O be the centre of Earth then Latitude of Karachi is the angle $\angle KOB$ denoted by φ and the Longitude of Karachi is $\angle AOB$ denoted by $\boldsymbol{L}$. If the place K $(\varphi, \boldsymbol{L})$ is in the upper hemisphere its Latitude is between 0^0 North and 90^0 North. If the place is towards East of the standard meridian then its Longitude is between 0^0 East to 180^0 East. The southern latitudes and the Western longitudes are defined in the same way.

2.3 ***CELESTIAL COORDINATE SYSTEMS***

There are a number of celestial coordinate systems that are essential for the understanding the computational astronomy. These are discussed briefly in the following:

2.3.1 GEOCENTRIC ECLIPTIC COORDINATES

The positions of points on the celestial sphere are defined in a number of ways. If the equatorial plane of the Earth is extended up to the sky, the celestial sphere, it intersects the sky in a great circle called the ***Celestial Equator***, as shown in Figure 3. For a person standing on the equator of Earth the celestial equator extends from due east to due west passing right above the head of the person through a point called ***Zenith***(the highest point in the sky). For an observer in the northern hemisphere the celestial equator also extends from due east to due west but remains south of the zenith. Similarly if the axis of rotation of Earth is extended to the sky it meets the celestial sphere in points P called the ***North Celestial Pole*** and the Q the ***South Celestial Pole***.

Although, it is Earth that revolves round the Sun, but relative to an observer on Earth the Sun travels around the Earth along a great circle called ***Ecliptic***. The planes of the Ecliptic and the Celestial Equator are inclined to each other at angle $\varepsilon = 23^0.5$, called the ***Obliquity*** of the Ecliptic. The two great circles, the ecliptic and the celestial equator, intersect in two points γ and γ', called the ***Vernal Equinox*** and the ***Autumnal Equinox***. In its "journey" around Earth, the Sun reaches γ on March 21, and γ' on September 21. γ, also known as, ***First Point of Aries***, and the Ecliptic form the reference of the Ecliptic Coordinate system. The point K on the celestial sphere, which is at 90^0 from each point on the ecliptic, is called the Pole of the Ecliptic. If S is any point on the celestial sphere then the let KSH be great circle arc joining K through S the ecliptic at H. The ***Ecliptic Latitude*** denoted as β, of S is the angle $\angle \boldsymbol{SOH}$ and the ***Ecliptic Longitude*** denoted as λ of S is the angle $\angle \gamma \boldsymbol{OH}$ (where O is now the centre of Earth). Thus (λ, β) are the ***Geocentric Ecliptic Coordinates*** of S.

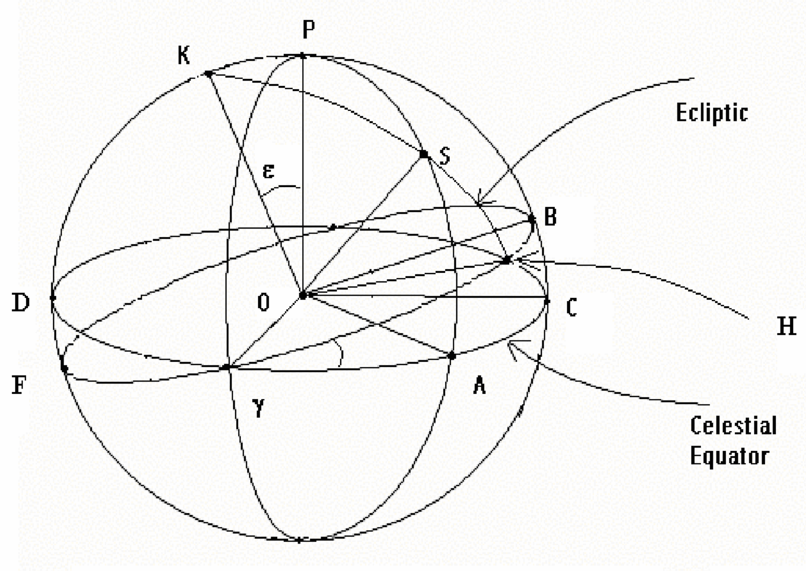

Figure 3

2.3.2 GEOCENTRIC EQUATORIAL COORDINATES

For the ***Celestial Equatorial Coordinates*** the references are the Celestial Equator and the Vernal Equinox, as shown in Figure 3 above. Let PSA is the great circle arc joining the North Celestial Pole, through the point S, the Celestial Equator at A. Then the angle $\angle SOA$ denoted as δ is called the ***Declination*** of S and the angle $\angle \gamma OA$ denoted as α is called the ***Right Ascension*** of S. Thus the ***Geocentric Celestial Equatorial Coordinates*** of S are (α, δ). Using the basic Formulas of Spherical Trigonometry the Ecliptic Coordinates of S and the Celestial Equatorial Coordinates of S are related by:

$$\boldsymbol{Sin}(\delta) = \boldsymbol{Sin}(\beta).\boldsymbol{Cos}(\varepsilon) + \boldsymbol{Cos}(\beta).\boldsymbol{Sin}(\varepsilon).\boldsymbol{Sin}(\lambda) \qquad (2.3)$$

and

$$\boldsymbol{Cos}(\alpha) = \boldsymbol{Cos}(\beta).\boldsymbol{Cos}(\lambda).\boldsymbol{Sec}(\delta) \qquad (2.4)$$

2.3.3 LOCAL EQUATORIAL COORDINATES

There is yet another celestial coordinate system, which is the most significant from practical point of view, shown in Figure 4. It is based on the position of the observer given by $(\varphi, \boldsymbol{L})$ on Earth. If an observer's latitude is φ then on his sky the North celestial pole P shall be at an Altitude same as φ. Thus the angle $\angle \boldsymbol{NOP} = \varphi$ where N is the North Cardinal Point on the observer's horizon. The Horizon of the observer, which is also a great circle, intersects with the Celestial Equator at points, E the east and W the West. The angle between the planes of the Celestial Equator and the Horizon of the Observer is $90 - \varphi$. The semi great circle joining the North (N) cardinal Point, the North Celestial Pole P, the Zenith Z and the South (S) cardinal point is called the Observer's Meridian. When any object is on the observer's meridian the object is said to be in Transit. After transit, the object, due to daily rotation of Earth moves towards the western sky.

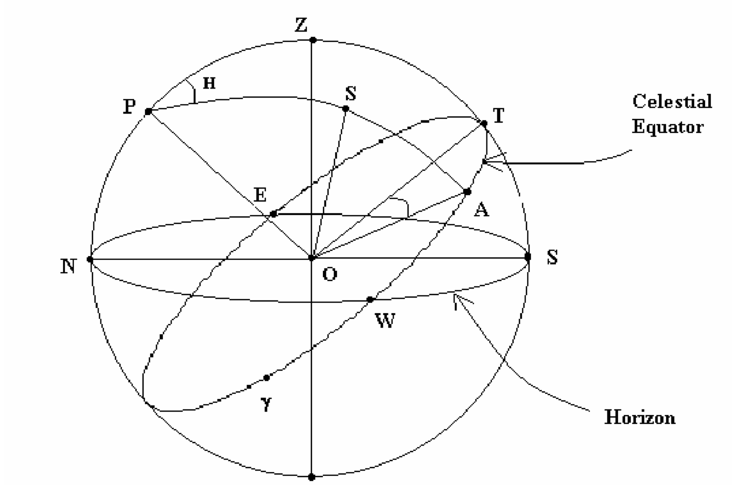

Figure 4

Let S is the object and PSA is the great circle joining the North Celestial Pole P, the object S and 'A' a point on the celestial equator. Then the spherical angle ZPS made between S and Z is called the Hour Angle of S (also equal to $\angle \boldsymbol{TOA}$, H being the point on the celestial equator that is highest in the sky of the observer). The Hour Angle is denoted as H. The $\angle \boldsymbol{SOA}$ is the declination of S. Thus the Local Equatorial Coordinates of S are $(\boldsymbol{H}, \delta)$. It should be noted that if γ is the vernal Equinox as shown in the figure 4, then $\angle \gamma \boldsymbol{OA} = \alpha$ is the right ascension of S, and that:

[Right Ascension (α) of S] + [Hour Angle (H) of S] = Hour Angle of γ (2.5)

This quantity, Hour Angle of γ, is called the Sidereal Time, which is extremely important in computational astronomy. This is so because actual time keeping in Astronomy is based on this time, and conversion from Celestial Equatorial Coordinates to the Local Equatorial Coordinates is not possible without knowing the exact "Sidereal Time".

Some of the telescopes have Polar Mounting. They have the advantage that scales show directly the Hour Angle and the Declination of the focused object. However, most of the amateur telescopes do not have this facility and have Alt-Azimuth mounting. The scales of these telescopes show the Altitude from Horizon of the object and the Azimuth of the Object. This highly localized coordinate system is described below:

2.3.4 ALTITUDE & AZIMUTH

The reference for this coordinate system is the local Horizon and the North Cardinal Point N as shown in Figure 5. This is most obvious of the coordinate system and the position of objects in sky can be easily observed on its basis. Consider the great circle arc ZSA. The Altitude of S is the angle $\angle \boldsymbol{SOA}$ made at the observer between the object and the horizon, generally denoted as ALT. Whereas the Azimuth is the angle $\angle \boldsymbol{NOA}$ measure along the horizon from North cardinal point towards the east. The azimuth is also given by 360^0 minus the spherical angle made at Z between the north celestial pole P and the object S.

Although the Altitude and the Azimuth are the most obvious of the of the coordinate systems, none of the physical theory directly describe the positions of planets and the Moon in their terms. The dynamical theories describe only the ecliptic coordinates which are then converted to the equatorial coordinate (α,δ) using formulas (2.3) and (2.4). Using (2.5) the Hour Angle is obtained. Finally the Altitude and Azimuth can be obtained using the formulas:

$$\boldsymbol{Sin}(\boldsymbol{ALT}) = \boldsymbol{Sin}(\varphi).\boldsymbol{Sin}(\delta) + \boldsymbol{Cos}(\varphi).\boldsymbol{Cos}(\delta).\boldsymbol{Cos}(\boldsymbol{H}) \quad (2.6)$$

and

$$\boldsymbol{Cos}(\boldsymbol{AZM}) = \frac{\boldsymbol{Sin}(\delta) - \boldsymbol{Sin}(\varphi).\boldsymbol{Sin}(\boldsymbol{ALT})}{\boldsymbol{Cos}(\varphi).\boldsymbol{Cos}(\boldsymbol{ALT})} \quad (2.7)$$

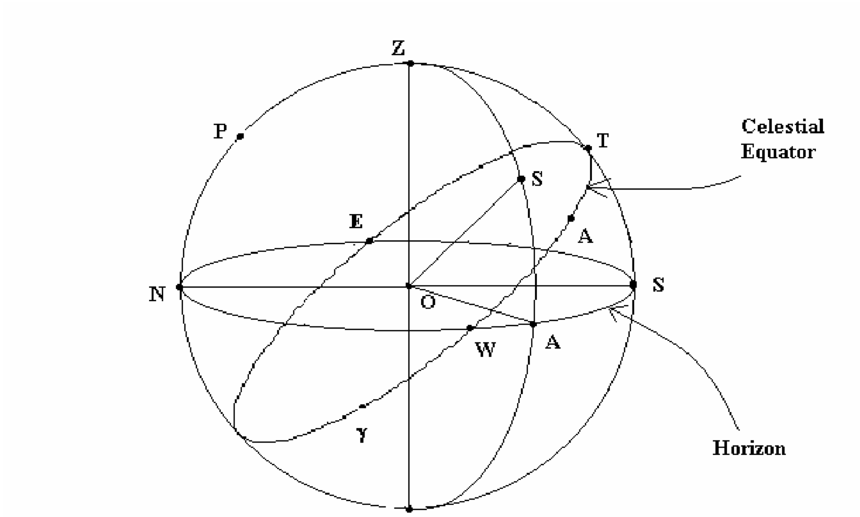

Figure 5

2.4 ***<u>THE TIME ARGUMENTS</u>***

The Time keeping methods[17] are of extreme importance in computational astronomy. The Astronomical time is the Sidereal Time which is the hour angle of vernal equinox γ. The Greenwich Mean Sidereal Time (GMST) is tabulated in Astronomical Almanac for 0 Hours U.T. for each date of a year. The vernal Equinox is not fixed and moves at a rate of 51' (arc seconds) per century. Besides, the period of one complete revolution of Vernal Equinox round the Earth is not constant. The Solar Day is defined generally as the duration of two consecutive transits of Sun. However this period is also not constant. The Sidereal Day is defined as the duration between two consecutive transit of γ. This period is not constant as well. In Computational Astronomy we talk of Mean Solar Day (the average over a year) and the Mean Sidereal Day. Mean Solar day is considered to be exactly 24 hours. Whereas the Mean Sidereal day is 3 minutes

56 seconds shorter than the Mean Solar Day. Thus the Astronomical Clock, the Sidereal Clock runs 3 minutes 56 seconds faster. One can obtain the Greenwich Mean Sidereal Time using the Formula:

$$\text{GMST at } 0^{hrs} \text{ U.T.} = 24110^{sec}.54841 + 8640184^{sec}.812866 * T_u + 0^{sec}.093104 * T^2_u - 6^{sec}.2 \text{ X } 10^{-6} * T^3_u \quad (2.8)$$

where $$T_u = (\text{JD} - 2451545.0)/36525 \quad (2.9)$$

and JD = is the Julian Date for the Noon at Greenwich on Januray 0, 2000. The Local Mean Sidereal Time (LMST) for a place with longitude ***L*** is then obtained by:

$$\text{LMST at } H^{hrs} M^{min} S^{sec} = \text{GMST} + \boldsymbol{L}(\text{in time measure}) + \text{Local Time} + (3^{min}\ 56^{sec}) * (\text{Local Time})/24 \quad (2.10)$$

Once the Local Mean Sidereal Time is obtained, the Hour Angle of any object can be obtained if the Right Ascension is known. The Julian Date for any date in the Gregorian Calendar may be obtained starting with: JD 1= Noon at Greenwich on November 24, for the year -4713. And the Gregorian rule of leap year:

The Y is a Leap year (29 days in February) if (Y is divisible by 4 or 400).
The Y is a not Leap year (28 days in February) if (Y is not divisible by 4 and 400).

3. ***<u>THE CELESTIAL DYNAMICS</u>***

The positions (coordinates) of the planets, the Moon and the Sun are determined using the principles of Celestial Dynamics. The principles of Celestial Dynamics required for most of the problems concerned here, are based on the Newtonian Mechanics. The coordinates of the Sun can be found using the simple Two-Body problem to a reasonable degree of accuracy. For the coordinates of Moon (and the planets) require a minimum of Three-Body problem and some unavoidable perturbations. In the following an account of the Two-Body problem and the methods of finding the coordinates of the Sun are discussed. This is followed by a brief account of methods to determine position of the Moon.

3.1 ***<u>COORDINATES OF THE SUN</u>***

According to Kepler's laws of Planetary motion and the Newtonian Mechanics the equation of orbit is given as follows, as considered in Two-Body Problem:

$$\boldsymbol{r} = \frac{\boldsymbol{a}(1-\boldsymbol{e}^2)}{1+\boldsymbol{eCos}(\boldsymbol{v}-\omega)} \quad (3.1)$$

where ***r*** is the heliocentric distance of planet, $\boldsymbol{e} < \boldsymbol{1}$ for an elliptic planetary orbit, ω is the longitude of the perihelion and ***v*** is the separation between the planet and the perihelion as shown in Figure 6. Along with this equation of orbit, the motion of the planet is completely described by the following equations:

$$\boldsymbol{n}^2\boldsymbol{a}^3 = \mu = \boldsymbol{G}(\boldsymbol{M}+\boldsymbol{m}) \quad (3.2)$$

$$\boldsymbol{E} - \boldsymbol{eSinE} = \boldsymbol{M} = \boldsymbol{n}(\boldsymbol{t}-\tau) \quad (3.3)$$

$$\boldsymbol{r} = \boldsymbol{a}(1-\boldsymbol{eCosE}) \quad (3.4)$$

and $$\boldsymbol{Tan}\left(\frac{\boldsymbol{v}}{2}\right)=\left(\frac{1+\boldsymbol{e}}{1-\boldsymbol{e}}\right)^{\frac{1}{2}}\boldsymbol{Tan}\left(\frac{\boldsymbol{E}}{2}\right) \qquad (3.5)$$

Where

$\boldsymbol{v}$ = the true anomaly or the separation of planet from the perihelion along the ecliptic,
$\boldsymbol{a}$ = the semi major axis of orbit, $\boldsymbol{G}$ is the Newton's Universal constant of gravitation,
$\boldsymbol{M}$ = the mass of the Sun,
$\boldsymbol{m}$ = the mass of the planet,
$\boldsymbol{n}$ = the mean angular velocity of the planet,
$\boldsymbol{E}$ = the eccentric anomaly or the angular separation of the planet from perihelion if it was moving in a circular orbit with radius equal to semi major axis

and τ = the time since the planet was last at the perihelion.

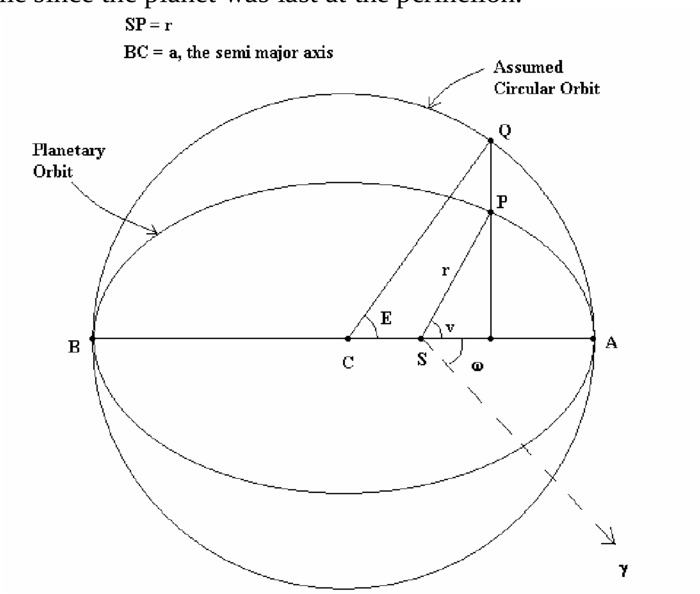

Figure 6

Together with $\boldsymbol{a}$, $\boldsymbol{e}$ and ω the following quantities are known as the ***Elements of the Orbit***:

Ω = Longitude of the ascending Node (which zero for Earth)
ι = Inclination of the orbital plane from the Ecliptic (which is also zero for Earth)

and $\boldsymbol{v}$ = the true anomaly or the angular separation of the planet from the perihelion.

For the motion of Earth around the Sun the Mean Longitude of the Sun at any time $\boldsymbol{t}$ is given by:

$$\boldsymbol{L}=\omega+\boldsymbol{M} \qquad (3.6)$$

Taking into account the deviation from this mean due to the elliptic nature of Earth's Orbit the following terms need to be added to (3.6):

$$\left(2\boldsymbol{e}-\frac{1}{4}\boldsymbol{e}^3\right)\boldsymbol{Sin}(\boldsymbol{M})+\frac{5}{4}\boldsymbol{e}^2\boldsymbol{Sin}(2\boldsymbol{M})+\frac{13}{12}\boldsymbol{e}^3\boldsymbol{Sin}(3\boldsymbol{M})+.. \qquad (3.7)$$

Higher order terms may be considered that appear in the solution of the Kepler's equation ($\boldsymbol{M}=\boldsymbol{E}-\boldsymbol{e}\boldsymbol{Sin}(\boldsymbol{E})$), but the third order expansions provide reasonably accurate results. The longitude of the Sun on the epoch J2000.0 was $280^0.46$, and the rate at which the Earth is going round the Sun is $0^0.985647359$ per day (from equinox to equinox), the Mean longitude of the sun is given by:

$$\boldsymbol{L}=280^0.46+0^0.985647359*\boldsymbol{n} \qquad (3.8)$$

The longitude of perigee of Earth's orbit on the epoch J2000.0 was $357^0.528$ and the rate at which the Sun is moving from perigee to perigee is $0^0.985600281$ per day the mean anomaly is given by:

$$\boldsymbol{M} = 357^0.528 + 0^0.985600281 * \boldsymbol{n} \qquad (3.9)$$

Thereby using $\boldsymbol{e}$ = 0.01669182 the third order expansion (3.7), the true longitude of the Sun can be calculated using the formula:

$$\lambda_s = \boldsymbol{L} + 1^0.915262268 * \boldsymbol{Sin}(\boldsymbol{M}) + 0^0.020008486 * \boldsymbol{Sin}(2\boldsymbol{M}) + 0^0.000289389 * \boldsymbol{Sin}(3\boldsymbol{M}) \qquad (3.10)$$

In both the formulas (3.9) and (3.10) $\boldsymbol{n}$ is the number of days since the epoch J2000.0 which can also be obtained form:

$$\boldsymbol{n} = \boldsymbol{JD} - 2451545.0 \qquad (3.11)$$

Without much of an error the Latitude β_s of the Sun can be considered zero as it remains on the ecliptic.

3.2 ***THE LUNAR ORBIT & ITS PERTURBATIONS***

The problem of the orbit of Moon must be considered at least as a Three-Body Problem with the Earth, E, the Moon, M, and the Sun, S as shown in the Figure 7. N being the ascending node of the Moon's orbit and the xy-plane representing the plane of the ecliptic. The orbit of the Moon around the Earth is much more complicated as compared to that of the Earth around the Sun. It is not only affected by the relative positions of the Earth and the Sun but it is also effected significantly by the relative position of the planets, particularly Venus and Mars. These effects are so significant that the orbital elements that are considered constants for the Earth are now all have both the secular variations as well as the periodic variations. In general the Lunar orbit is a highly irregular ellipse whose plane is inclined to the plane of ecliptic at an angle of around $5^0.15$ that varies periodically up to ± 9 arc minutes. The eccentricity of this "elliptic" orbit is 0.0549 but varies up to ± 0.0117. The Ascending Node of the lunar orbit moves along the ecliptic and completes one revolution in about 18.6 years.

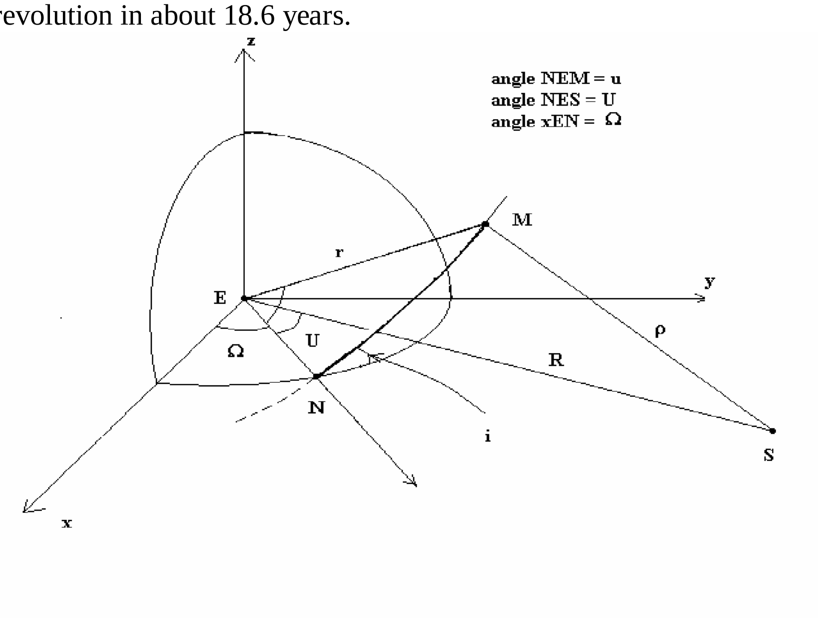

Figure 7

The perihelion moves along the lunar orbit and completes one revolution in about 8.85 years. Besides these secular motions of the ascending node and the perihelion, there are small range periodic variations that affect the position of Moon. The ascending node oscillates about $\pm 1^0.67$ and the perihelion oscillates about $\pm 12^0.33$. Due to these large scale and the small scale variations in the orbital elements the position is calculated on the basis of the average rates of variation of every quantity over a centaury. A detailed account of the same shall be explored in a separate work. In the following, for the sake of brevity, the formulas of low precision are considered in this work:

$$\lambda_m = 218.32+481267.883*T + 6.29*SIN(134.9+477198.85*T) -1.27*SIN(259.2-413335.38*T) +0.66*SIN(235.7+890534.23*T) +0.21*SIN(269.9+954397.7*T)-0.19*SIN(357.5+35999.05*T) -0.11*SIN(186.6+966404.05*T) \quad (3.12)$$

$$\beta_m = 5.13*SIN(93.3+483202.03*T) + 0.28*SIN(228.2+960400.87*T) -0.28*SIN(318.3+6003.18*T)-0.17*SIN(217.6-407332.2*T) \quad (3.13)$$

Where T is the number of centuries since J2000.0, that may also be calculated using (2.9). The position of Moon as it appears in our sky is greatly affected by the Parallax that also has a periodic variation and is given by:

$$\pi = 0^0.9508 + 0^0.0518\boldsymbol{Cos}(134^0.9 + 477198^0.85*\boldsymbol{T}) + 0^0.0095\boldsymbol{Cos}(259^0.2 - 43335.38*\boldsymbol{T}) + 0^0.078\boldsymbol{Cos}(235^0.7 + 890534^0.23*\boldsymbol{T}) + 0^0.0028\boldsymbol{Cos}(269^0.9 + 954397^0.7*\boldsymbol{T}) \quad (3.14)$$

The width of the Moon as it appears in our sky is also important in our context so we require the semi-diameter SD of the Moon that is given by the relation:

$$\boldsymbol{SD} = 0.2725*\pi \quad (3.15)$$

This concludes all the tools that we require for the computational effort for determining position of the Sun and the Moon in our sky and the way the Lunar crescent appear their.

4. *VISIBILITY OF NEW LUNAR CRESCENT*

The criteria for the visibility of the new lunar crescent are based on local coordinates (altitude and azimuth) of the Moon and the Sun. For the local time of observation (after Sunset on a day after conjunction), one should calculate the ecliptic coordinates of each of them using (3.10), (3.12) and (3.13). Using these Ecliptic coordinates of the Sun and the Moon Right Ascension and Declination of each of them is determined using (2.3) and (2.4). Using (2.5) and the local sidereal time the Hour angle is determined. Finally using (2.6) and (2.7) the altitude and azimuth of the Sun and the Moon are found for an observer at the terrestrial coordinates $(\varphi, \boldsymbol{L})$. In these calculations, appropriate Local Sidereal Time and the Julian Date for the time of observation should be employed, that are discussed in article (2.4).

4.1 *THE ANCIENT METHODS*

All the ancient methods[1] of determining whether the new Lunar Crescent shall be visible at any place consider the ***Lag***. This is the interval between the Sunset and the moon set. This means that since times of Babylonians the man understood the motion of heavens to high degree of precision and had fairly accurate time keeping methods. Besides Lag the Babylonians considered the ***Age of Moon***. This is the time elapsed since the last ***Conjunction*** (when the ecliptic longitude of the

Sun and the Moon coincide). In literature it is reported that Babylonians considered the following condition for the New Lunar Crescent to be visible at any place:

(i) ***Age > 24 hours*** (ii) ***Lag > 48 minutes*** (4.1)

The Muslim/Arab astronomers modified this criterion to the following:

(i) ***Altitude > 8 degrees*** (ii) ***Lag > 45 minutes*** (4.2)

Under perfect visibility conditions, that depends on the atmospheric conditions, (4.2) is still a reliable criterion for visibility of crescent. Later observational work has shown that the crescent has been seen with un-aided eye at an age as early as 12 hours and 36 minutes[19-24]. However the recorded scientific observation has no incidence of visibility of crescent when the separation between the Moon and the Sun is less than 8 degrees[29], which is known as the Danjon Limit[5][6]. Whereas the shortest Lag reported is 38 minutes and 18 seconds when the crescent was seen with unaided eye. Earlier sightings of crescents, both in terms of age and Lag, have been reported with binoculars and telescopes.

4.2 ***THE MODERN APPROACH***

A number of 20th century astronomers have contributed towards the development of a mathematical model for the criterion of earliest visibility conditions on the new lunar crescent. The works of Maunder[13], Schoch[25] and Bruin[3] are of fundamental importance. These works are mostly based on the observational records of the past two centuries. The earliest excellent observational records are due to Schmidt[13] who made his observations of the crescent (both the morning and the evening crescent) during the last 20 years of the 19th century.

4.2.1 MAUNDERS METHOD:

Each methods used to develop a prediction criterion is based on an effort to determine the region of sky around the point where the Sun sets which is bounded by quadratic or a cubic curve. If the crescent happens to be within this region it would not be visible. Due to inclination of the ecliptic with the local horizon and the inclination of the lunar orbit with the ecliptic, the position of the crescent when it can be seen just after Sunset may vary both in azimuth and altitude on either side of the point of Sunset. Using the recorded observations Maunder[13][29] considered the following optimum values of relative azimuth (DAZ = difference between the azimuths of the Sun and the Moon) and the altitude (ARCV) when the crescent is seen with un-aided eye:

DAZ	**0°**	**5°**	**10°**	**15°**	**20°**
ARCV	**11·0**	**10·5**	**9·5**	**8·0**	**6·0**

Then using Least Square approximation he fitted this data to a quadratic polynomial to obtain ***ARCV*** as the following function of ***DAZ***:

$$\boldsymbol{ARCV} = 11 - \frac{|\boldsymbol{DAZ}|}{20} - \frac{\boldsymbol{DAZ}^2}{100} \qquad (4.3)$$

The plot of this function is shown in the Figure 8. This can be interpreted as follows: If the new lunar crescent happens to be on or above the Maunders curve (in western sky) at the time of sunset at any place, it would be visible to the naked eye other wise not. According to Maunders method the crescent would be visible to naked eye if the altitude is more than that given by (4.3) for the relative azimuth on the day of observation.

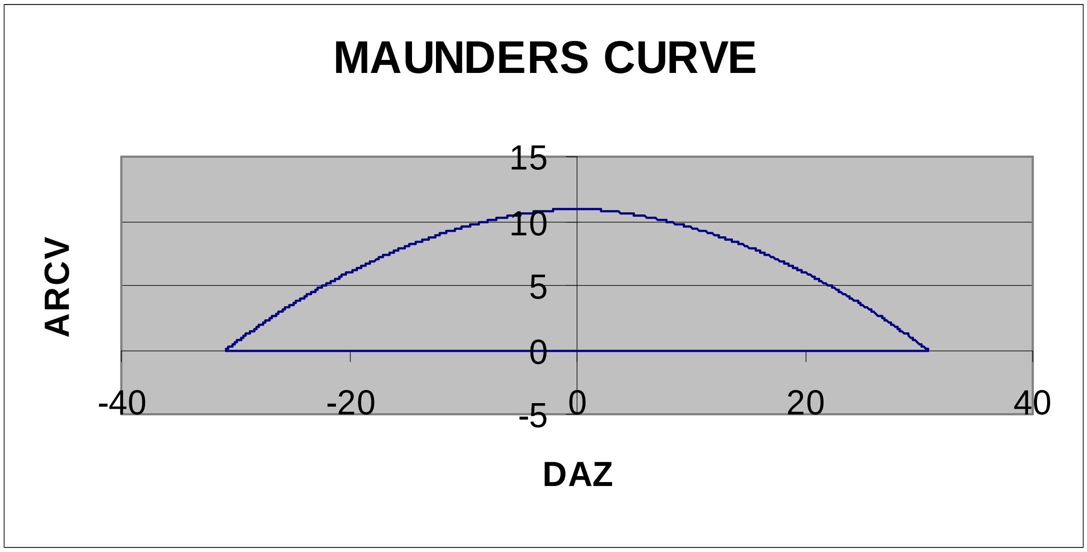

Figure 8

4.2.2 THE INDIAN METHOD:

The method developed on the basis of Schoch[25] is known in literature as the Indian method[11] as the data used by Schoch has been adopted by the Indian Astronomical Ephemeris since 1966. This basic data is:

DAZ	**0°**	**5°**	**10°**	**15°**	**20°**
ARCV	**10·4**	**10·0**	**9·3**	**8·0**	**6·2**

This data when fitted to a second degree polynomial using Least Square Approximation yield the following relation between DAZ and ARCV:

$$\boldsymbol{ARCV} = 10.3743 - 0.0137|\boldsymbol{DAZ}| - 0.0097(\boldsymbol{DAZ})^2 \qquad (4.4)$$

which is correct to 5 decimal places. The maximum in the quadratic Indian Curve is less than that in the maunders Curve as it appears in the Figure 9.

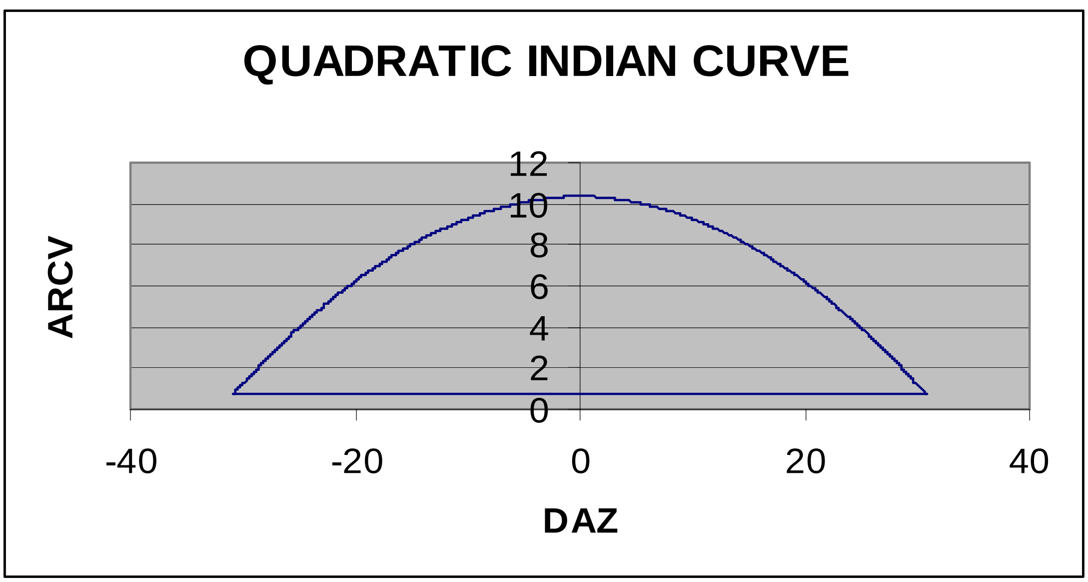

Figure 9

So according to Indian method the crescent would be visible to the naked eye if its altitude is greater than that given by (4.4). The Indian quadratic curve is lower than the Maunders curve, thus it predicts earlier visibility of crescent as compared to the Maunders method.

4.2.3 ***BRUINS METHOD & YALLOP'S CRITERION:***

Both the Maunders and the Indian method have considered the altitude of the Moon as the key factor in the earliest visibility of the new lunar crescent. However apart from the Altitude it is the width of the crescent at the time of observation that constitute an important factor as it is directly related to the brightness of the Moon. The central width of the crescent is a measure of the ***Phase*** of the Moon. Phase of the Moon is a function of the separation between the Sun and the Moon and is defined as the part of the lunar disc illuminated at any time. The separation between the Sun and the Moon is known as ***Elongation*** or ***Arc of Light*** (***ARCL***). In terms of ***ARCL*** the phase of Moon is given by:

$$\boldsymbol{Phase} = \frac{1-\boldsymbol{Cos}(\boldsymbol{ARCL})}{2} \qquad (4.5)$$

It is reported in the Astronomical Almanac[27] that the Lunar crescent is visible to the naked eye when its Phase becomes greater than 0.01, but much depends on the visibility conditions. This condition allows a minimum separation of 11.47 degrees between the Sun and the Moon in order that the crescent becomes visible. In literature, the minimum ARCL reported is 8.5 degrees when the crescent was seen with naked eye. Bruin[3], considering the importance of Phase considered the observational data about the actual visible width ***W*** of the central part of crescent in arc minutes and the ***ARCV*** to develop a relation between the two as the criterion for earliest visibility of crescent. The data he used was:

W	0'·3	0'·5	0'·7	1'	2'	3'
ARCV	10·0	8·4	7·5	6·4	4·7	4·3

Fitting a third degree polynomial to this data the function obtained is:

$$\boldsymbol{ARCV} = 12.4023 - 9.4878\boldsymbol{W} + 3.9512\boldsymbol{W}^2 - 0.5632\boldsymbol{W}^3 \qquad (4.6)$$

Bruins approach is more realistic especially in view of the fact that due to varying distance and speed of the Moon the lunar width (measured in terms of ***Semi Diameter*** **SD**) as it appears in our sky varies. This variation is also not negligible for different places on the Earth, which is due to Parallax. During the year 2004 the maximum semi diameter of the Moon was 16.7 arc minutes and the minimum was 14.7 arc minutes which is almost 13% of the average. As the ***ARCL*** is related to both ***DAZ*** and ***ARCV*** by:

$$\boldsymbol{Cos}(\boldsymbol{ARCL}) = \boldsymbol{Cos}(\boldsymbol{ARCV}).\boldsymbol{Cos}(\boldsymbol{DAZ}) \qquad (4.7)$$

and the width ***W*** is related to ***ARCL*** by:

$$\boldsymbol{W} = \boldsymbol{SD}.(1-\boldsymbol{Cos}(\boldsymbol{ARCL})) \qquad (4.8)$$

Where ***SD*** is given by (3.16). Thus one can easily convert (4.6) into a relation between ***ARCV*** and the semi diameter ***SD*** plus the ***ARCL***. Instead, Yallop has used a version of the Indian Method in terms of the width ***W*** of lunar crescent at the time of observation thus taking into account the parallax. In an attempt to develop a single parameter test for the visibility of crescent, he defines a ***q-value*** as:

$$\boldsymbol{q} = (\boldsymbol{ARCV} - (11.8371 - 6.3226\boldsymbol{W} + 0.7319\boldsymbol{W}^2 - 0.1018\boldsymbol{W}^3))/10 \qquad (4.9)$$

On the basis of this q-value test Yallop[29] studied the recorded observations over the past 150 years and developed the following criteria for the visibility of the new lunar crescent:

(A)	$q > +0{\cdot}216$	Easily visible ($ARCL \geq 12°$)	V
(B)	$+0{\cdot}216 \geq q > -0{\cdot}014$	Visible under perfect conditions	V(V)

(C)	$-0{\cdot}014 \geq q > -0{\cdot}160$	May need optical aid to find crescent	V(F)
(D)	$-0{\cdot}160 \geq q > -0{\cdot}232$	Will need optical aid to find crescent	I(V)
(E)	$-0{\cdot}232 \geq q > -0{\cdot}293$	Not visible with a telescope *ARCL* ≤ 8·5°	I(I)
(F)	$-0{\cdot}293 \geq q$	Not visible, below Danjon limit, *ARCL* ≤ 8°	I

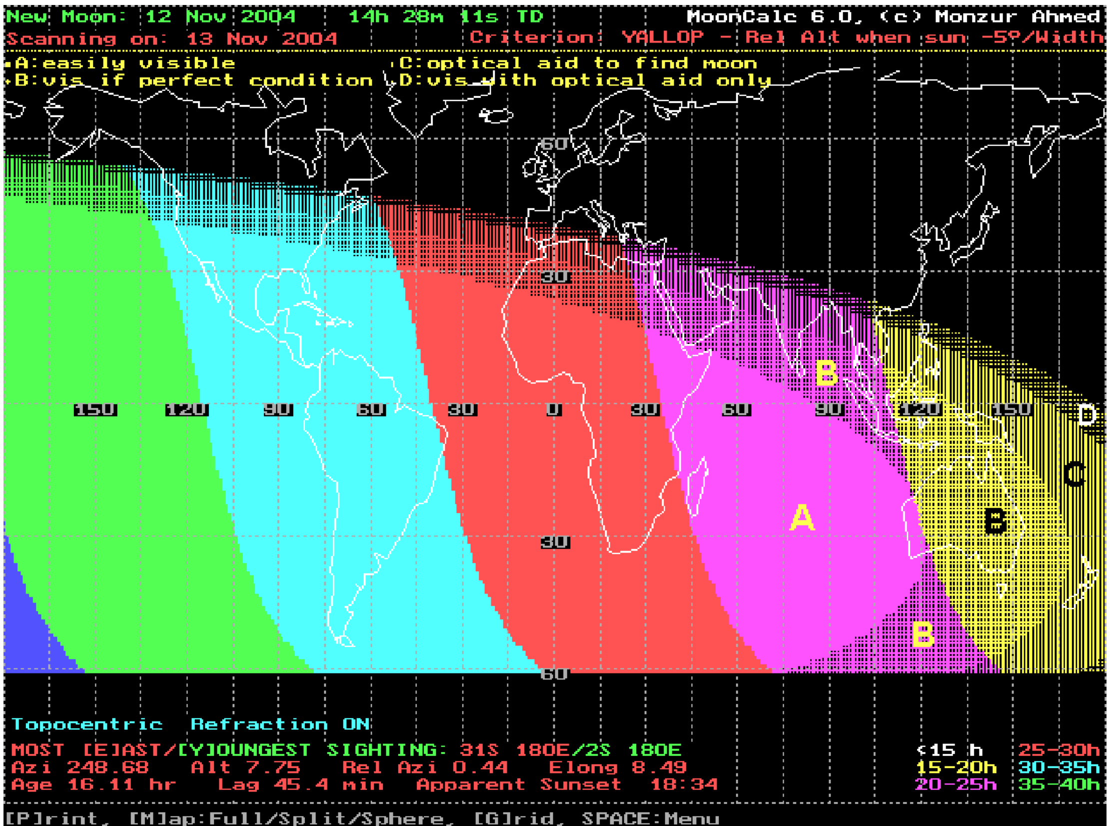

Figure 10. The "Moon Map" developed using MoonCalc by Ahmed[1].

The q-value 0.216 defines a pseudo-parabolic curve on the globe (surface of Earth) with the characteristic that in all places to the west of the curve the new lunar crescent is easily visible with naked eye. This curve is called curve A. The q-value -0.014 also defines a pseudo parabolic curve to the west of the curve A and is called the Curve B. In t region between curve A and the curve B t lunar crescent may be visible to un-aided eye under perfect visibility conditions. The q-value -0.16 define the curve C such that in the region between Curve B and the curve C the crescent require a binocular or a telescope to first locate the Moon and then under perfect visibility conditions it can be seen with naked eye. The q-value -0.232 defines the curve D such that between Curve C and the Curve D the crescent may be seen only with the aid of a binocular or a telescope. In the regions to the East of curve D the crescent can not be visible even with an optical aid. The map for the beginning of the current lunar month (after conjunction at 14:28 GMT, on November 12, 2004) is reproduced here from a software mentioned in the next article.

5. ***HIGHER ORDER CURVE FIITING:***

In this work higher degree polynomial are fitted to the data in each of the three methods discussed in the previous article using Least Square Approximation. For Maunders method the second degree curve fitting produced a result that is identical to (4.3). Whereas fitting a third degree polynomial resulted in a second degree polynomial:

$$\boldsymbol{ARCV} = 10.99999 - 0.049996|\boldsymbol{DAZ}| - 0.010001\boldsymbol{DAZ}^2 \qquad (5.1)$$

This suggests that instead of fitting a second-degree curve to that data using least square approximation Maunder actually reached (4.3) empirically. The result obtained (5.1) fits the data, used in Maunders method, more closely.

For the data in Indian method the second degree curve fitting with coefficients correct upto 6 decimal places obtained is:

$$\boldsymbol{ARCV} = 10.374287 - 0.013714|\boldsymbol{DAZ}| - 0.009714(\boldsymbol{DAZ})^2 \quad (5.2)$$

Whereas as third degree polynomial fitted to the same data yields:

$$\boldsymbol{ARCV} = 10.394279 - 0.04237|\boldsymbol{DAZ}| - 0.005716(\boldsymbol{DAZ})^2 - 0.0001333|\boldsymbol{DAZ}|^3 \quad (5.3)$$

The Visibility Curve obtained on the basis of (5.3) is shown in the following figure:

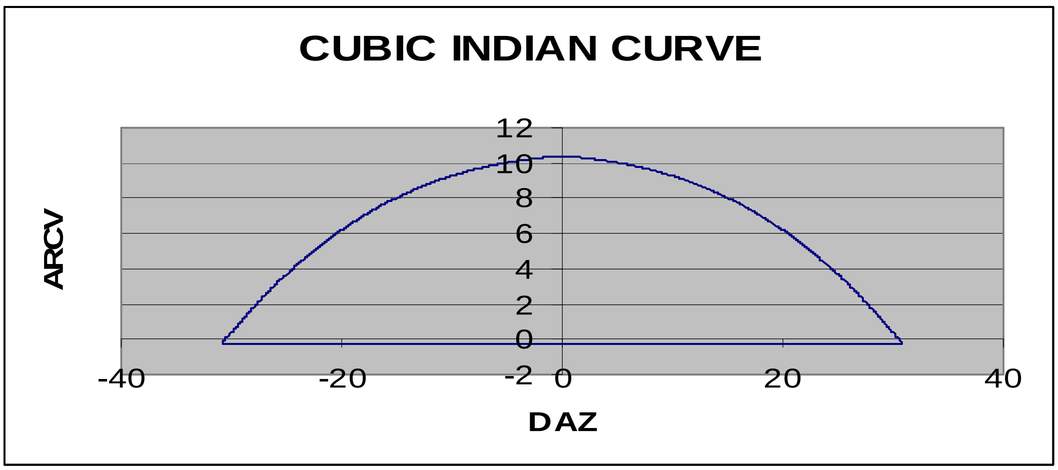

Figure 11

This curve is as symmetrical as the Maunders Curve about the Vertical from the point of sunset but its maximum is below the maximum of the Maunders Curve and marginally above the maximum of quadratic Indian curve. However it is still consistent with the fact that Indian Curve implies the earliest visibility of crescent at altitudes less than that indicated by the Maunders Method. Finally the third degree polynomial fitted to the data of the Bruin's method resulted into the following function:

$$\boldsymbol{ARCV} = 11.514933 - 7.503203\boldsymbol{W} + 2.74385\boldsymbol{W}^2 - 0.348139\boldsymbol{W}^3 \quad (5.4)$$

The coefficients obtained are significantly different from those of (4.10) but the function fits the data more closely as compared to (4.10).

On the other hand, from the data of the Indian method two tables are generated in this work one for maximum value of ***W*** = 16.7 arc seconds and another for the minimum value of ***W*** = 14.7 given below:

SD = 16.7

W	0'·27436	0'·316	0'·4699	0'.726	1.099'
ARCV	10.4	10.0	9·3	8·0	6·2

SD = 14.7

W	0'·24	0'·28	0'·4136	0'.64	0'.97
$ARCV$	10.4	10.0	9·3	8·0	6·2

These tables are generated using (4.11) and (4.12). The third degree polynomial fitted to these two data sets yield the following two cubic functions:

$$ARCV = 12.190506 - 8.743478W + 5.032535W^2 - 2.478246W^3 \qquad (5.5)$$

and $$ARCV = 12.180283 - 7.637586W + 3.802054W^2 - 1.64261W^3 \qquad (5.6)$$

Both these results are markedly different from what is reported by Yallop. Moreover, (5.5), (5.6) and the function used by Yallop in (4.9) values of altitude of Moon, ***ARCV,*** as W is increased beyond 2 arc minutes, that are negative. However the original data of Bruin and his expression (4.6) give positive values to ***ARCV.*** It is so because the expression (4.6) due to Bruin is based on actual data whereas (5.5), (5.6) and (4.9) are only simulated results that are based on an estimation technique. In the Table 1, some of the critical observations reported by various authors and observers and arranged and analyzed by Yallop are shown with q-values due to Yallop, Bruin and ccalculated in this work on the basis of (4.9), (5.5) and (5.6) are presented.

NO.	DATE	Lat	Long	ARCL	ARCV	DAZ	PHS	PAR	W	YAL	BRU	SH-B	SH-Y1	SH-Y2	VISIB.
		DEG	DEG	DEG	DEG	DEG			MIN						CODE
164	28.01.1979	37.8	-122	11.9	11.7	2.3	0.0107	61.4	0.361	0.205	0.223	0.255	0.213	0.186	V
155	09.03.1978	50.3	-119	12.7	11.5	5.4	0.0122	58.1	0.389	0.202	0.222	0.251	0.209	0.181	V
100	29.01.1922	-34	18.5	19.5	8.8	-17	0.0287	54	0.846	0.185	0.194	0.188	0.191	0.136	I
28	24.06.1865	38	23.7	18.6	8.7	16	0.0261	57.1	0.815	0.158	0.171	0.167	0.163	0.111	I
99	30.12.1921	-34	18.5	18	9.1	-16	0.0245	54.7	0.731	0.154	0.174	0.174	0.158	0.111	I
204	31.01.1911	51	-0.9	16.4	9.7	13	0.0203	56.6	0.629	0.158	0.184	0.191	0.164	0.123	V
2	27.10.1859	38	23.7	21.4	6.8	20	0.0345	57.5	1.082	0.108	0.075	0.063	0.132	0.052	V
230	19.01.1988	32.2	-111	12.2	9.7	7.4	0.0113	61.3	0.378	0.016	0.035	0.065	0.023	-0.005	I(V)
36	12.05.1869	38	23.7	13.5	9.1	10	0.0138	56.4	0.426	-0.02	0.006	0.031	-0.01	-0.039	I
220	05.11.1983	37.2	-84.1	13.1	8.9	9.6	0.013	57.8	0.411	-0.05	-0.02	0.003	-0.04	-0.067	I(V)
246	24.04.1990	41.6	-73.7	12.1	9.1	-7.9	0.0111	61.1	0.371	-0.05	-0.03	8E-04	-0.04	-0.069	I(V)
10	05.01.1861	38	23.7	20.2	5.5	19	0.0308	61.1	1.026	-0.05	-0.07	-0.083	-0.03	-0.107	I
46	06.07.1872	38	23.7	11.4	9.6	6.1	0.0099	53.9	0.291	-0.05	-0.04	0.004	-0.04	-0.064	I
196	26.06.1987	42.7	-84.5	10.5	9.5	4.4	0.0084	54	0.247	-0.08	-0.08	-0.032	-0.08	-0.1	V(B)
157	27.01.1979	35.2	-112	10.4	9.2	-4.8	0.0082	61.4	0.276	-0.09	-0.09	-0.045	-0.09	-0.113	V(B)
44	14.09.1871	38	23.7	9.3	8.9	2.6	0.0066	57.1	0.205	-0.17	-0.17	-0.119	-0.17	-0.186	V
294	20.01.1996	34.1	-118	8.9	8.8	-1.6	0.006	61.2	0.201	-0.18	-0.18	-0.131	-0.18	-0.198	I(V)
278	25.02.1990	35.6	-83.5	8.5	8.5	-0.6	0.0055	59.4	0.178	-0.22	-0.23	-0.176	-0.23	-0.243	V(V)
169	31.12.1986	39	-77	12.4	6	11	0.0117	61.3	0.39	-0.35	-0.33	-0.298	-0.34	-0.368	I(B)
194	25.06.1987	-30	-71	9.6	4.2	8.6	0.007	54.3	0.207	-0.64	-0.64	-0.587	-0.64	-0.654	I(B)
195	26.06.1987	-30	-71	9	4	-8	0.0062	54	0.181	-0.67	-0.68	-0.624	-0.68	-0.691	I(B)

TABLE 1

6. ***SHORT COMINGS OF THE PREDICTION CRITERIA***

On basis of various prediction criteria for the earliest visibility of New lunar crescent the surface of Earth is generally divided into two regions by the pseudo parabolic curve mentioned in the last section (the curve A of Yallop). The same is done on the basis of both the ancient and the modern criteria. Ilyas[8-10] called this curve the ***International Lunar Date Line*** (***ILDL***), and defines it as the curve where the probability of sighting the new lunar crescent is 0.5. Ahmed[1] has prepared excellent software called MoonCal whose version 6 is available on the net. This software develop the ILDL on the basis of 18 different criteria including the Yallops criterion. The maps produced by MoonCal are considered most reliable.

The major draw back with these maps is that they are made for perfect visibility conditions that are not available at all in the heavy populated cities and their vicinity. Their real test can be observations from places that are far away from optical and other type of pollutions. So predictions made on the basis of these maps are definitely not reliable for city observers. Moreover the data that has been collected by Shaukat[27] and is regularly reported on the website www.moonsighting.com contains naked eye sightings that violates the Yallops detailed criterion. These observations are reported in this work along with the calculation of q-values based on Yallops criterion in Table 2. The observations that are below the critical limit of naked eye observation are considered as EXTRA ORDINARY.

CITY	DATE	ARCV	DAZ	ARCL	SD	W	Q-value	REPORT		COMMENTS
LONG BEACH(USA)	14.03.02	8	7.85	11.2	14.9	0.2833	-0.21025	I		REQUIRED OPTICAL AID
SAN FRANS.	14.03.02	7.5	7.25	10.42	14.9	0.2456	-0.2827	B	N	EXTRA ORDINARY
MOMBASA	15.03.02	15.6	3.7	16.02	14.9	0.5788	0.719676	N		EASILY VISIBLE
DAMASCUS	15.03.02	14.25	7.67	16.15	14.9	0.5877	0.589621	N		EASILY VISIBLE
TUCSON	13.04.02	8.55	8.07	11.74	15.1	0.3146	-0.13671	N		EXCELLENT OBS.
DAMASCUS	14.04.02	17.91	7.61	19.41	15.1	0.8554	1.100915	N		EASILY VISIBLE
SIGNAL HILL	13.05.02	7.07	4.9	8.595	15.8	0.1774	-0.36677	N		EXTRA ORDINARY
GEORGE TOWN	13.05.02	15.19	1.45	15.26	15.8	0.5569	0.666441	N		EASILY VISIBLE
SAN JOSE	12.05.02	4.5	1.3	4.684	15.8	0.0528	-0.70055	I		IMPOSSIBLE
KUWAIT	11.07.02	12.9	5.5	14.01	16.5	0.4905	0.399989	N		EASILY VISIBLE
AMMAN	09.08.02	9.63	6.41	11.55	16.7	0.3372	-0.01542	N		GOOD OBSERVATION
KUWAIT	07.09.02	6.55	3.3	7.331	16	0.1305	-0.44739	I		IMPOSSIBLE
TRINDAD	07.09.02	10.2	2.15	10.42	16	0.2633	-0.00213	N		GOOD OBSERVATION
KUWAIT	06.10.02	2.67	1.07	2.876	17	0.0214	-0.9032	I		IMPOSSIBLE
NEBRASKA	06.10.02	4.7	4.6	6.573	17	0.1114	-0.64416	I		IMPOSSIBLE

Table 2

Besides, observations reported by Schaeffer as a result of a number of Moon Watch programs suggests that 15 percent sightings were simply illusions. This high percentage of error is also reported on the basis of the available visibility criteria.

The public sightings reported by Official committees of some counties for religious purposes are frequently found highly deviated from these visibility criteria. There seems to be no operational method to verify these claims. Any observational astronomer can not reach every place where sighting criteria are below the critical limits and where a public claim is available.

6. ***CONCLUSIONS***

Computationally none of the criteria can be perfect. All computational techniques are based on some sort of averages as discussed in sections 4.1 and 4.2. The actual dynamics of the Sun and the Moon deviates from these averages. Therefore, the estimation of the position of the Sun and the Moon in terms of celestial Latitude and the Longitude is always in error. The errors in case of the Sun may reduced very small values but those of the Moon can never be made so small that we can clearly define limiting criterion for the earliest visibility of crescent. The limiting criteria are all probabilistic. Therefore, deviations from predictions or astronomical criterion and observations are always possible. In this work it has been found that the calculations of Yallop are inconsistent. The fitted polynomial on the basis of width of crescent is realistic but a criterion on the basis of $\boldsymbol{W}$ requires more observed data in limiting cases so that relations between $\boldsymbol{ARCV}$ and $\boldsymbol{W}$ may improved. Further, for the development of better criteria for earliest crescent sighting it is recommended that a large number of observatories should make observation of crescent just before and after the conjunction so that a sizable amount of data is collected for all sorts of q-values and thereby an improved criteria is deduced.

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