

# Moonset Lag with Arc of Light Predicts Crescent Visibility

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## **ABSTRACT.**

I investigate the question of crescent visibility relying on modern data. The specific focus is a promising criterion for visibility when viewing by naked eye or binoculars (as later specifically defined), although telescopic observation will also be taken into consideration. The criterion makes use of the moonset lag (delay between sunset and moonset), together with auxiliary input based on the arc of light (angular separation between the sun and moon).

**Key words:** Moon — visibility

## **Introduction**

An extensive literature and lore exists about how to achieve greatest success at sighting the crescent. Even assuming that all due preparations have been undertaken to seek out the clearest conditions, to know accurately where and when to look, and optionally to make effective use of optical technology, it can also be an asset to have a good advance concept of the level of detection difficulty. From a scientific standpoint, and in the context of a very specific mode of observing, when can one make a prediction that visibility will be either (1) normally possible, or (2) so marginal and dependent on unmeasurable factors as to be irreducibly uncertain, or (3) guaranteed to be impossible (for that mode of attempted sighting)?

Odeh(2006), Hoffman(2005), Caldwell&Laney(2000), and Yallop(1997) give thorough contemporary overviews of the issues involved with developing such lunar visibility prediction criteria, so that an exhaustive exposition of the subject can be forgone here. In essence, for the crescent to be perceived, its illuminance has to be humanly detectable as contrasting sufficiently with the brightness of the adjacent sky. Scholars and scientists have parameterized (i.e. computationally modeled in terms of useful indicators) the circumstance in which the crescent can have attained that illuminance threshold after conjunction (waxing moon), or the symmetrical boundary case before conjunction (waning moon), by experimenting with the use of a variety of indicators. These include the moon age since conjunction, the time lag between the moon and sun rise or set, the detectable width or circumferential extent of the crescent, the angle of separation between the sun and moon from the geocentric perspective using known laws of gravity (referred to as the arc of light), and the altitude and azimuth displacement angle between the sun and moon from the “topocentric” viewpoint of the observer.

By employing one or a combination of several such parameters, the above authors and others have aspired to devise a criterion that dependably gauges the emergence of the crescent from

an invisible to a visible status. The critical threshold value for the criterion is calibrated by (i.e. matched to the outcomes of) past sightings of record. Besides yielding a reliable judgment as to sighting possibility, a secondary desideratum is for a criterion to be straightforwardly calculable, on a location by location basis, by easily available means, as well as sufficiently comprehensible to the educated public that its misconstruction and resulting disputations are unlikely. Caldwell&Laney(2000) identified that, under very specific provisos, the moonrise/set lag [hereafter, lag] as a function of the arc of light, defined above, offers particular promise when applied to naked eye [hereafter, visual] and binocular sighting. The crescent visibility insight from the lag versus the arc of light is the subject of this study.

## Provisos

Crescent visibility study has a large following and the field is flourishing in diverse and novel directions as technology and enterprising techniques have developed. There are traditional twilight visual sighting, sighting with binocular, by telescope, by telescope-coupled ccd (i.e. charge coupled device) camera, and by telescope/ccd-coupled computerized differential imaging. There is airborne as well as ground-based. Besides twilight crescent sighting there is also peri-solar midday optically-aided viewing. [Hereafter, twilight will be discussed in terms of dusk, although dawn is equally intended when appropriate]. Telescopic, ccd, airborne and daytime-style sighting are hereinafter explicitly excluded unless cited in a particular context. For acquaintance with these modes not analyzed here, the website <http://www.icoproject.org/record.html> is a commendable starting point.

The societally relevant lunar calendar enjoins the pursuit of visual crescent sighting. Binocular visibility thus also retains direct relevance since it is the sighting mode extending the perception grasp one simple step beyond the visual technique, as well as often being the pointing aid for the visual attempt to follow. The other sighting modes (airborne, midday, computer-processed, etc.), though promising and fascinating, do not in any obvious way contribute more understanding of the visual observability than afforded simply by direct perusal of the visual plus binocular results on their own terms. I will classify as a binocular-level (as opposed to telescopic-level) event, any attempted sighting with an optical device of aperture 150mm or less, whether double-barrel or not (for example “finder” or “spotting” scopes of compatible aperture would belong), whereas any larger aperture device will be classed as telescopic and thus excluded.

## Event Collection

Data selection or exclusion is an important issue in science, and the danger of post-facto omission to support a prejudice (even unconsciously) must be avoided by clearly setting up the “selection rules” independently of the analysis itself. The current data set (appendix A and B) is foundational for this study and consists of 36 positive visual and 58 positive binocular sightings. These numbers may appear to be surprisingly small, but this is because in a typical lunar cycle, sighting the young crescent at any given location is normally either fairly easy or completely impossible. The marginal cases that define the limits of visibility are relatively rare.

The sources of the basic data for the sighting events come from the published literature,

together with the online data base of crescent sighting results at the Islamic Crescent Observation Project (<http://www.icoproject.org/>)[hereafter, ICOP]. Many published literature sources may be conveniently accessed at ICOP as well, and therefore this paper forgoes citing a vast bibliographical foundation for the synoptic and historical background of the subject. Important specific sources for the current sighting events include: Odeh(2006), Hoffman(2005), Hoffman(2003), Caldwell&Laney(2000), Yallop(1997), and Ilyas(1994). Lastly, the extensive fund of sightings [hereafter, “sighting” encompasses both positive and negative sighting attempts as appropriate] reported worldwide and logged on an ongoing basis at ICOP was carefully studied to bring the available sighting data up to date.

Some exclusions have been applied as follows. It was decided to exclude sightings prior to 1980. This does not disparage earlier achievements but more modern results are increasingly numerous, while many older results are relatively harder to find out much background detail for. A relatively high fraction of older recorded sightings tend to involve crescents too unchallenging to be of interest in this work. The very sparsity of the early results coming down from several generations past, gives rise to some concern about their homogeneity with the contemporary body of consistently executed and well-reported sightings. A lag cutoff of 45 minutes was imposed, as sightings of the crescent where the lag is greater than this value are generally too easy to aid in defining the limits of visibility.

Some previous analyses have opted to synthesize both sightings and nonsightings as complementary guidance. The useful interpretation of non-sightings is however encumbered by distracting extraneous effects that they interject, for example about either local meteorological or putative observer procedural hindrances. The present study will hold off summarizing the import of negative sightings until a stage where the interpretation of the inherently more clearcut information from the positive sightings has first been assimilated.

## Questions

Natural questions at this point are: what is a sighting really, and why turn to this set of sighting reports and not others?

Difficult crescent sightings such as collected and analyzed here differ notably from a non-practitioner’s likely concept of happening to notice the crescent. A sighting by a skilled practitioner generally follows much of the following template, with the maximum effort by eye and brain. (a) Attention is focused on the right track of sky at the right clock time. (b) For roughly two minutes preceding true visibility, stimuli of a local brightening are perceived up to several times, strengthening into a perception of a brightening with frustrating location and completely uncertain shape. Reporters use such terms as fluctuations or glimpsings. (c) Location becomes rather secure but shape remains elusive. (d) True visibility arrives when location and shape perception, and repeatable finding looking away and then back, are attained.

Experienced crescent observers, at the edge of perception bordering from case (c) to (d) must do their best in a contest with their own objectivity, as well as fight off eye fatigue from prolonged staring, which conceivably could induce spurious impressions. The burden for objectivity cuts both ways, to be willing to see what your espoused criterion may subconsciously influence you to fail to see, and to be willing to concede a non-sighting, that you subconsciously

worry could cast doubt on your previously secured sightings. Experienced observers can also gauge the self-deception danger from wisps of cloud or edges in haze layers. Isolated well-delineated cloud bodies in the same part of the sky need not hinder a sighting claim, but a deceptive “blue band layer” that supposedly penetrates a very cloud-mottled horizon sky should almost without exception invite observer skepticism.

The current post-1980 data set includes all available published or at ICOP, excepting some extremely few in journals that increasingly these days are no longer shelved in academic libraries, as well as being unobtainable online. They are in my opinion collectively representative of a high threshold of experience, a fair threshold of carefulness and objectivity, and an additional element which is a degree of peer scrutiny resulting from the logging or critiquing/publishing process. The observing conditions of these particular observations were by and large good to excellent, particularly near the visibility boundaries to be discussed. Sightings that went unpublished, or unlogged, could not be used for lack sufficient original details when read about in later undetailed second-hand accounts. This will not bias our results if the reason their details remained unavailable is not an effect of the cause being that the adequate details of these rumored sightings were prevented from reaching a published or online-logged form for the very reason of their implications being discordant with the body of sightings which did manage to have their adequate details appear in available formats. In short we posit that our data set is not biased because of prejudicial omissions.

## Data Tabulation

Appendices A and B give the data tabulation form of the 36 visual and 58 binocular positive crescent sighting events, respectively. The tabulation was filled out starting with the key source information, and then adding values calculated with the MICA Multiyear Interactive Computer Almanac (<http://www.usno.navy.mil/USNO/astronomical-applications/software-products/mica>) from the USA Naval Observatory. Columns 1-3 give the Universal Time [hereafter, UT] year, month, and day of the sunset or sunrise associated with the event of that line in the table. Note that this can validly differ from the (also valid) civil date appearing in the raw report, for some (especially western N. American) longitudes and seasons.

Column 4 is the lag in minutes (see under column 12-15 below). The moonset lag is the difference between time of sunset and time of moonset. During this interval the twilight sky grows dimmer, boosting the contrast between the sky and the lunar crescent. Greater lag also correlates with more leeway to sight the crescent *before* its becoming screened by the very high opacity atmospheric layer that occupies the few degrees starting right at horizon level. Columns 5-8 give the observer longitude and latitude in degrees and minutes. In a few cases of multiple nearby observer locations, a simplified joint figure was used for the location, because the small geographical spread has negligible effect on the really important quantities, viz. the lag and the arc of light.

Columns 9-11 give the UT day, hour, and minute of the geocentric conjunction (New Moon) corresponding to the other facts on the same line. Columns 12-13 give the UT sunset (or sunrise) moment in hours and minutes. Columns 14-15 give the UT moonset (or moonrise) moment in hours and minutes. The lag in column 4 was initially calculated as the difference, which can, about 1 time in 4 (depending on precision), give rise to a 1 minute round-off error. Although

$\pm 1$  minute is minor, we enhanced the calculation to the time-seconds level of precision for any lag figure of 30 minutes or less. (The very few cases where column 4 differs from the inference of columns 12-15 by one minute, denote such adjustments.) The rise/set times include the factors of refraction, semi-diameter, and topocentric parallax, but no adjustment for site elevation. As it is only the difference of the times that matters in the lag context, and as the appropriateness and method (absolute or only relative to the surrounding topography) of applying elevation corrections is very context dependent, I concluded that there no justifying the complication of using individualized elevation adjustments.

Columns 16-19 are the astrometric geocentric (equator J2000) right ascension and declination of the sun, in hours, minutes, degrees, and arc minutes, at the mid-lag moment between sunset and moonset. Columns 20-23 correspondingly give the sky position for the moon. Finally column 24 gives the arc of light [hereafter, arcl], namely the separation of sun and moon center gauged at the mid-lag time, using

$$\cos(A) = \cos(90 - B) * \cos(90 - C) + \sin(90 - B) * \sin(90 - C) * \cos(15 * (D - E)) \quad (1)$$

where A is the arcl, B and C are the declinations of the sun and the moon, in decimal degrees, and D and E are the right ascensions of the sun and moon, in decimal hours. As the arcl is not a large angle in this context, the planar geometry approximation

$$(A)^2 = (B - C)^2 + (15 * (D - E) * \cos((B + C)/2))^2 \quad (2)$$

is sufficiently accurate for the purpose: the median and the maximum absolute deviation from the rigorous results are only  $0.003^\circ$  and  $0.041^\circ$ , respectively. The arcl from physics principles bears a nearly one-to-one correspondence with the continuous degree of intrinsic dimming and brightening that the crescent undergoes as its orbit returns to and then departs from conjunction, accompanied by the decrease and then increase of its illumination phase angle.

Since the arcl continuously varies, its use as a parameter requires a decision about the appropriate moment to takes its measure. That moment should logically coincide with the carrying out of the sighting attempt for which the arcl is supposed to provide a part of the criterion. A standardized evaluation moment must be adopted, and the estimated moment of easiest visibility (Sultan 2006 and references therein) is a leading choice. Another is the estimated time of first visibility (Hoffman 2005). This study uses the mid-lag moment, but in fact any standardized point of time in the range from sunset to moonset is suitable as long as it is used consistently across all the events that are being intercompared on an equal footing. No single choice really stands superior, because the practical effect of all of them is merely small shifts to the arcl parameter scale, which is being used as only a relative indicator, not for its precise quantitative figure.

Another rather purist consideration that turns out to be immaterial has to do with the almanac coordinate system for expressing the sun and moon celestial sky position. There exist subtly distinct coordinate system choices that have to do mostly with valid alternatives for specifying the origin (“zero zero”) and principal plane of each system. However, since the only physical reality that matters is the great circle angular separation between sun and moon, and not the detailed figures of the coordinates (except for scholarly cross-checking purposes) which after all amount merely to a man-made bookkeeping system, it is again a case of clearly setting out and consistently employing a single reasonable coordinate basis, rather than the choice of

which coordinate basis.

Topocentric times are relevant for observer witnessed events like rising and setting, but I chose to use geocentric measures of the growth of the moon's brightness hour-by-hour because the effects proceed more uniformly without adding topocentric-incurred scatter, for a superior one-to-one correspondence with intrinsic brightening function (Caldwell&Laney 2000). Another minor clarification is that for some events the declination equals zero degrees minus some arc minutes, i.e. referring to a sky position less than one degree south of the celestial equator. For such cases I have set the southern declination minus sign by the arc minutes column. This is the computer-friendly remedy for preventing declination expressions using “-00”, which most software cannot correctly read as input.

## First Look

Fig. 1 shows the result of plotting the lag at these positive sighting events versus the arcl of the same. Black squares are visual sightings while x points are binocular. Given the 30-plus years of collection baseline, the data yield seems surprisingly modest. There are two explanations. Firstly, there do actually exist many more positive visual sightings than shown here, but they without exception cluster at larger lag than these critical events. Most sightings occupy a preferred swath with lag longwards of 45 min and arcl in the range 10-15°. That class of very ordinary, numerous, “run-of-the-mill” sightings is precisely the kind I intentionally bypassed by selecting a sample that probes the limits to visibility. A second factor is that a significant fraction of the reported attempts falling into the plotted lag:arcl range turned out to be non-sightings, which abundantly occupy the zone directly below that of the points plotted here, plus a sprinkling upward where nonvisibility also was the outcome, but most probably for meteorological reasons, since comparably difficult cases were positively sighted at other events, viz. the ones plotted. My deliberate choice of paring to just the most solid clues lies behind the plot appearance.

In terms of gross trends, there is an absence of sightings with low lag, yet there also appear to be clear exclusion zones toward the upper left and lower right of the diagram. With the exception of five visual points on the lower left, there seems to be a fairly clear but modest trend of smaller lag sightings becoming enabled by compensatingly larger arcl. There are also two binocular sightings standing off at low lag from the data consensus. I will argue below that the five visual and two binocular outliers should be interpreted separately, and so putting those aside, we concentrate on the lay of the visual and binocular sightings relative to each other. In terms of the reported sighting procedures, it is as a rule true that any binocular positive sighting is identically a visual negative sighting: had visual sighting succeeded, the report would have been processed into the visual positive (black squares) category.

It was notable in some of the report descriptions how the observers at events pushing the lower lag and lower arcl boundaries drew attention to the extreme and, for each individual, unaccustomed level of difficulty at that achieved sighting event. Both from such comments and also from the bands where binocular(Yes)/visual(No) points lie, it seems clear that for some small value of the lag, which depends partly on arcl, even with excellent conditions and observing prowess, the eye simply lacks the requisite degree of stimulus to trigger a real perception response. The light amplification provided by binoculars allows the eye to perceive the crescent

at a somewhat more challenging visibility level. Progress beyond this first cut at interpretation requires simulating some features of the lag:arcl plane as well as further scrutinizing the outlier sighting points.

## Closer Look

Fig. 2 shows the result of taking a closer look at these data and also running some simulations of the lag and the arcl for arbitrary values of the observer latitude. I start by discussing the vertical dotted band with arcl value  $7.0-7.5^\circ$ . This is the Danjon Limit (Danjon 1932, 1936) which has been thoroughly discussed by Sultan(2007), Fatoohi etal(1998), and Schaefer (1991, 1993). It sets a minimum arcl for crescent visibility because the length of the visible crescent (from cusp to cusp) shrinks as the arcl decreases. In Schaefer’s words, “the shortening [is] a natural consequence of the crescent’s rapid brightness decrease towards the cusps” and all these works confirm nonvisibility by an arcl cutoff of about  $7.0-7.5^\circ$ .

Turning now to the five exceptional visual sightings at low lag and arcl, marked as “?”, is there any way to infer why they are different? Indeed all these, and only these, data come from a sole source, a citation by Ilyas(1994) of a publication by Qurashi(1991), which is said to contain a series of reports of record-setting Pakistani sightings. Unfortunately the actual Qurashi reports were not available for this current study, only the Ilyas citation of them. I will refer to the Qurashi-supplied sightings in a later section but set them aside unused in the analysis section of this paper.

Turning to the two exceptional binocular sightings around lag 20, is there any way to infer why they are different? Indeed they, and only they, used a special new technique as follows. The usual binocular sightings use careful pointing (sometimes with theodolite assistance) and a slight amount of small-amplitude scanning for the crescent during nearly the full lag duration. The two exceptional binocular sightings, as they were very thoroughly reported at ICOP, include novel features. (1) The single best visibility moment (from theory) for the entire lag interval is pre-calculated. (2) The crescent altitude-azimuth location at that future moment is pre-calculated. (3) The top grade binocular equipment has its pointing calibrated on the night preceding the attempt, and is locked into position. By using the nighttime sky as a pointing reference, there is absolute assurance about the pointing validity, unlike customarily where one sets up in the daylight sky just preceding the attempt, and uses pre-tested equipment gauges or local horizon landmarks, to get close to (within about a degree of) the correct altitude-azimuth track of the crescent descending to the horizon. (4) Finally the sighting is carried out with a burst of effort only at the ideal couple of minutes of best opportunity. Even then, the crescent was reported to have taken maximum effort to see, and to be unlikely to be ever viewable with any further incremental difficulty burden whatsoever.

So far we know that visibility is blocked, firstly, at low arcl and, secondly, at low lag for a typical arcl, but what about very large arcl? We know that large lag (but moderately low arcl) sightings are extremely common (the ones not shown in this work with lag longwards of 45 min and arcl  $10-15^\circ$ ), while on the other hand, high arcl (but low lag) sightings are extremely uncommon (in fact no reported instances). Key facts about the lag are that it depends roughly speaking on the moon’s altitude at sunset and its rate of travel toward the horizon. The latter depends on the diurnal angular velocity (proportional to the inverse cosine of the moon’s

declination) and upon the moon's diurnal angle of descent to the horizon (proportional to the inverse sine thereof), where the angle of descent  $F$  is given by

$$\cos(F) = \sin(\textit{latitude}) / \cos(\textit{declination}) \quad (3)$$

(Smart 1962, p.53). The altitude at sunset that the moon attains depends on its celestial path after conjunction. That path is determined by the moon's orbit, which is inclined by  $5.15^\circ$  relative to the ecliptic (the plane of the earth's orbit around the sun). The ecliptic in turn is at an angle of  $27.43^\circ$  to the celestial equator (the projection of the earth's equator onto the sky). An added complication is that the points where the moon's path crosses the ecliptic change considerably with time. There are also relatively small but complex changes in the lunar orbit itself, but these are not vitally important here.

Thus the moon's celestial path since conjunction, as seen from a viewpoint near the earth's equator, subtends a range of possible angles  $\pm 32.6^\circ$  from the vertical. At middle to high earth latitude, that  $65^\circ$  spread in the range of possible moon elongation directions, is tilted toward the south in the northern hemisphere, or towards the north in the southern hemisphere, and so the crescent altitudes being compressed toward the horizon, moonset lags are consequently decreased. Contrariwise the descent angle, which is  $90^\circ$  at the equator regardless of declination, from the above formula also becomes compressed to shallow angles from a middle-to-high latitude perspective, depending on the moon declination within its circumscribed range. For high enough latitude the descent angle goes to zero as the moon declination reaches deeply enough into the contrary hemisphere. The lags are therefore increased as the descent angle wanes, so that simply put, high latitude can work both ways influencing the lag.

To understand the lag and arcl effects better, a simulation was run of all moon elongation ascent (viz. the celestial path taken post-conjunction) and diurnal descent angles for the full year pattern, at choices of latitude, starting from conjunction up to the a maximum of 48 hours accumulated elongation, which encompass the first two moonsets following conjunction, plus some margin. The current study sighting data have a very marked (absolute) latitude distribution [hereafter, southern and northern are discussed as merged since the physics is symmetrical]. The distribution has a median absolute latitude of  $32^\circ$ , a first quartile of  $31^\circ$ , and a third quartile of  $33^\circ$ , with but few extending to the equatorial and to the temperate zones. None is at high latitude. It appears that the statistically clear weather, milder or less severe twilight outdoor conditions relative to higher latitudes, and cultural factors have combined to give the achievement of champion category sightings an overwhelming latitude imprint.

Now we can better understand the lay of the data in the diagram. For low to moderate latitude ( $0-35^\circ$ ), the elongation and descent angles range over all possibilities such that for a small lag, no large arcl is possible, and correspondingly for a small arcl, no large lag is possible. This simulated range is shown in Fig. 2 by the two diagonal lines with crossed dashes running up their length. The actual data fill between the said lines, except as expected the sightings lapse both lower down and leftwards to the Danjon boundary. For higher latitude ( $45-55^\circ$  was used for contrast and because population and hence conceivable sighting activity decline rapidly beyond that) very large lags are enabled by very low descent angle, while very low lag is in principle also enabled by very low ascent angle of elongation. This second simulated range is shown by the left boundary dotted with circles and the bottom right bottom along the lag = 0 axis. The low lags that can potentially result emphasize that for geographically middle to high latitudes, very different sighting geometries with descent angles much compressed to the

horizontal prevail, in contrast with the customarily familiar sightings from latitudes below  $40^\circ$  with their characteristically substantial descent angles.

When the arc of light has a full 48 hours to grow (assuming only the first two post-conjunction moonsets are relevant), what prevents seeing crescents down to nearly zero lag? The property is one known as “phase space,” meaning that while such an undertaking from high latitude vantage is not formally impossible, the range of possibilities is such that a very large boost to the lag is statistically the prevalent outcome from a large ensemble of large arcl crescent sightings as pursued from high latitude. To concoct the viewing geometry for a sighting in the far lower right regime of Fig. 2 requires excessively fine-tuned special conditions, because most stochastically transpiring scenarios will undergo a substantial boost in the lag time. The likelihood to land in the favored lag  $\geq 45$  sighting zone overwhelms the likelihood of getting the combination of diurnal descent angle and elongation ascent angle just right so as to witness an event exhibiting high arcl jointly with low lag.

## Visual Visibility Based on Lag and Arc of Light

Fig. 3 summarizes the gist of the evidence amassed. The pattern of sightings as a group reinforce the impression of a visibility drop-off at low lag, but increased arcl definitely plays a facilitating role. Disparate observing conditions (principally different haziness) have the asymmetric character that poor conditions eventuate in every degree up to “super-poor,” but excellent conditions have no corresponding extension to “super-excellent” because the aerosol, smoke, dust, ash etc. load in the air, for a given geographical location, can attain only a limiting air quality and not beyond (on a human not geological time scale). Therefore it is justified to identify the observed cutoffs of this sighting ensemble as illuminating the threshold of “the best you can realistically expect to do.” You could encounter slightly worse visual sighting conditions (viz. the overlap zone of visual and binocular sightings) due to slightly variable haziness or any combination of small (ungaugeable) probabilistic effects (a “zone of uncertainty” which is expected near any visibility transitional boundary). However decades of experience justify concluding that one does not anticipate valid visual sightings far surpassing the celestial (lag,arcl) difficulty of these shown. It bears re-emphasizing that numerous negative sightings by reliable observers (see sources in Introduction) repeatedly and abundantly affirm that crescent visual/binocular invisibility is axiomatic in the low-lag, low-arcl quadrant of the data plot, in full consistency with the positive sightings analyzed here.

To summarize the boundaries, visibility criterion lines are suggested as follows. Line (A) numerically traces a visual sighting boundary. Line (B) does the same for the generally practiced binocular sighting method. These become dashed when extrapolated into the lower-lag, higher-arcl, low-phase-space (increasingly improbable chances of such an occasion being obtained, proceeding to the lower right, as explained above). Criteria such as these, when calculated on the globe for each lunation, generate lunar date line curves that divide the globe into positive prediction and negative prediction zones. Even in favorable weather and air quality, it is unavoidable that there be an interleaving geographical zone of uncertainty within which the prediction loses decisiveness (becomes a 50/50 guess) because of the un-gaugeable nature of the process right at the borderline of visibility. The specially executed binocular viewing at low lag (lowest diamond) is a reasonable anchor for the extreme uncertainty range of the criterion. At lower arcl than that, I expect it will be harder to achieve as much added visibility grasp despite

the massive, team-intensive, input of added preparation and execution labor, while at larger arcl little is known or will easily become known. I denote that guessed zone of ignorance with a dashed line (C) with a nod to the Qurashi input; all dashed lines are merely speculative whereas only the solid lines have real, though tentative, support, awaiting the slow fruition of more such labors. The equations for lines (A), (B), and (C) are

$$\begin{array}{rcll}
 \text{(criterion A)} & \text{lag} & = & -0.9709 \text{ arcl} + 44.64 \\
 \text{(criterion B)} & \text{lag} & = & -1.4150 \text{ arcl} + 45.88 \\
 \text{(criterion C)} & \text{lag} & = & \begin{array}{l} \text{the larger of} \\ \text{or} \end{array} \begin{array}{l} -1.9230 \text{ arcl} + 43.13 \\ -1.4150 \text{ arcl} + 36.76 \end{array}
 \end{array}$$

where the interpretation is: visual sighting possible to (A), improbable to (B), impossible below; binocular sighting possible to (B), improbable to (C), impossible below.

## Conclusions

I have summarized the real evidence that, as expected from physical principles, true sighting grasp of the crescent must lapse for unfavorable enough celestial (low lag, low arcl) circumstances, specifically with reference only to the ground based, twilight, visual or binocular method. However to decide on the basis of lag, the arcl must be taken into account as well, and the criteria presented here supply a guideline for that.

Additional insights gained while undertaking this work can also be mentioned here, mostly in nature of opinions rather than established facts. (1) Twilight telescopic observations probably yield positive sighting potential in the indicated uncertainty zone marked (C) but not much below that. Examples are slowly accruing but remain too sparse to proceed beyond such an anecdotal assessment. (2) It is important not to confuse the public with informational attempts that mix the various sighting modes together willy-nilly as this quickly confuses the issue of what predicted visibility outcomes can rationally be expected, distinguished according to method. (3) With any large enough scale undertaking, there is normally a residual of false positives that are immune to being discounted by any available indications for doubt. In the light of this, one approach is that “Extraordinary claims require extraordinary proof” ([http://en.wikipedia.org/wiki/Marcello\\_Truzzi](http://en.wikipedia.org/wiki/Marcello_Truzzi) etc), which could be interpreted to mean that a digital or analog recording of such sightings is a requirement. This is a high bar to set and rather selectively dismissive tack, especially since normally the full detailed description supplied by the observer is taken as adequate input on its own. A fairer recourse would seem to be to collect the fullest possible information documenting such very divergent “super-observations” on a probationary and separate basis, pending confirmatory repeat findings. Where digital or analog proof is lacking, and the observations fall into the low-lag zone where many other observers have reported negative results, especially where the only available source is second-hand (the Qurashi points), one can then justifiably classify such input as unsubstantiated and expired in terms of the next large-scale and comprehensive synthesis reviewing the field.

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## FIGURE CAPTIONS

**Figure 1.** The moonrise/set lag (viz. lag) versus the arc of light (viz. arcl) for positive visual sightings (squares) and positive binocular sightings (x points).

**Figure 2.** As Fig. 1 but also with Danjon's Limit (dotted vertical band), simulated possible range of lag:arcl for absolute latitude  $0-35^\circ$  observers (bracketed by the two diagonal lines with cross-dashes), and for absolute latitude  $45-55^\circ$  observers (filling in the rectangle above and right of the two lines with dotted circles).

**Figure 3.** Three criterion lines (A), (B), (C) as described in the text, give interpretation to the crescent visibility implications from combining the moonset lag and the arc of light as indicators.

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# Appendix A

## 36 Critical Positive Visual Crescent Sightings in 24 Data Columns

01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1987	04	28	31	71	24	34	01	28	01	34	13	55	14	26	2	22	14	10	2	43	18	46	6.82
1989	04	06	29	73	05	33	40	06	03	33	13	32	14	00	1	2	6	38	1	19	12	31	7.22
1989	10	01	32	34	43	31	24	29	21	47	15	26	15	58	12	32	-3	24	13	36	-14	52	19.50
1990	02	25	39	-83	30	35	36	25	08	54	23	25	00	04	22	36	-8	51	23	2	-3	25	8.44
1990	04	25	29	74	13	31	20	25	04	27	13	38	14	07	2	12	13	16	2	27	19	50	7.48
1990	04	25	31	73	05	33	40	25	04	27	13	45	14	16	2	12	13	16	2	27	19	51	7.50
1991	01	16	25	73	05	33	24	15	23	50	12	23	12	48	19	52	-20	57	20	16	-19	24	5.84
1993	02	23	39	18	25	-33	55	21	13	05	17	30	18	09	22	28	-9	36	23	49	4	9	24.42
1996	01	21	34	18	25	-33	55	20	12	50	17	58	18	32	20	13	-19	55	21	18	-11	5	17.96
1996	10	13	41	34	39	31	48	12	14	14	15	11	15	52	13	16	-8	3	14	7	-10	28	12.81
1997	02	08	33	18	25	-33	55	07	15	06	17	46	18	19	21	30	-14	47	22	28	-7	23	16.03
1997	05	07	39	34	39	31	48	06	20	46	16	26	17	05	2	59	16	58	3	47	15	22	11.63
1997	08	04	35	35	13	31	46	03	08	14	16	33	17	08	8	59	17	5	9	55	10	30	15.10
1998	02	27	38	18	25	-33	55	26	17	26	17	25	18	03	22	42	-8	13	23	37	-3	47	14.37
1998	04	25	38	51	24	35	36	26	11	41	01	50	01	12	2	10	13	5	0	58	2	58	20.47
1999	05	14	40	34	39	31	48	15	12	05	02	46	02	06	3	22	18	30	2	9	7	53	20.67
1999	05	14	40	35	31	31	48	15	12	05	02	42	02	02	3	22	18	30	2	9	7	53	20.67
1999	05	14	40	37	06	31	42	15	12	05	02	36	01	56	3	22	18	30	2	9	7	51	20.69
2000	01	07	34	18	25	-33	55	06	18	14	18	01	18	35	19	13	-22	23	19	59	-20	5	10.96
2000	04	03	36	35	13	31	46	04	18	12	03	25	02	49	0	50	5	24	23	41	-6	22	20.86
2000	07	31	36	03	24	06	30	01	19	20	18	05	18	41	8	45	18	3	9	25	17	23	9.55
2000	09	28	44	34	53	29	38	27	19	53	15	30	16	14	12	21	-2	19	13	9	-1	54	12.00
2000	12	26	42	20	49	-32	23	25	17	22	17	46	18	28	18	23	-23	20	19	12	-22	31	11.31
2001	03	25	36	18	25	-33	55	25	01	21	16	51	17	27	0	19	2	2	0	54	0	24	8.90
2001	07	21	43	73	18	04	06	20	19	44	13	23	14	06	8	4	20	23	8	51	20	40	11.01
2002	03	12	33	51	24	35	36	14	02	03	02	50	02	17	23	28	-3	27	22	14	-15	50	22.01
2002	03	15	43	-111	06	32	13	14	02	03	01	32	02	15	23	39	-2	16	0	26	-2	26	11.74
2003	01	31	37	51	42	32	36	02	10	48	03	29	02	52	20	53	-17	32	19	49	-24	53	16.61
2003	09	27	41	-111	00	32	24	26	03	09	01	16	01	57	12	13	-1	24	13	5	-3	40	13.18
2003	10	26	42	50	10	33	15	25	12	50	13	54	14	36	14	2	-12	25	15	1	-16	50	14.93
2004	01	22	44	03	42	32	30	21	21	05	17	08	17	52	20	17	-19	43	21	10	-21	35	12.53
2004	09	15	39	35	00	31	53	14	14	29	15	46	16	25	11	35	2	45	12	26	0	-20	13.11
2004	12	13	41	-110	56	32	26	12	01	29	00	19	01	00	17	22	-23	9	18	23	-27	56	14.55
2006	07	26	44	-104	01	30	41	25	04	31	01	55	02	39	8	21	19	31	9	7	20	13	10.84
2008	12	28	44	51	42	32	36	27	12	12	13	36	14	20	18	31	-23	15	19	22	-23	58	11.70
2010	10	09	29	-104	01	30	41	07	18	44	00	33	01	01	12	57	-6	6	13	56	-16	40	17.89

(01-03)	year,month,day of SS or SR	(12-13)	hour,min of SS or SR	(20-21)	R.A. of moon in hour,min		
(04)	lag from SS to MS or MR to SR	(14-15)	hour,min of MS or MR	(22-23)	decl. of moon in ° ' ,		
(05-08)	longitude ° ' , latitude ° ' ,	(16-17)	R.A. of sun in hour,min	(24)	arc of light		
(09-11)	day,hour,minute of conjunction	(18-19)	decl. of sun in ° ' ,	SS =	sunset	MS =	moonset
				SR =	sunrise	MR =	moonrise

# Appendix B

58 Critical Positive Binocular Crescent Sightings in 24 Data Columns

01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1983	11	05	44	-84	06	37	12	04	22	21	22	35	23	19	14	43	-15	48	15	37	-17	33	13.05
1984	11	23	38	-81	00	34	00	22	22	57	22	17	22	55	16	0	-20	34	16	55	-24	13	13.22
1989	05	06	42	-97	00	30	18	05	11	46	01	08	01	50	2	53	16	32	3	20	23	30	9.42
1991	03	17	44	-110	42	32	24	16	08	10	01	31	02	15	23	46	-1	33	0	12	6	29	10.33
1995	01	02	43	-106	00	33	00	01	10	56	00	08	00	51	18	49	-22	58	19	20	-17	30	9.09
1996	01	21	41	-118	04	34	13	20	12	50	01	10	01	51	20	10	-20	5	20	36	-13	40	8.93
1997	03	08	40	34	39	31	48	09	01	15	04	01	03	21	23	15	-4	53	22	24	-7	43	12.98
1997	05	07	36	50	48	36	00	06	20	46	15	29	16	05	2	59	16	58	3	45	15	16	11.18
1997	05	07	37	51	42	32	36	06	20	46	15	19	15	56	2	59	16	57	3	44	15	15	10.94
1997	12	30	34	18	25	-33	55	29	16	56	18	00	18	34	18	40	-23	8	19	35	-17	43	13.97
1998	01	27	41	-95	22	29	46	28	06	01	13	14	12	33	20	39	-18	25	19	57	-17	6	10.08
1999	09	10	44	-77	00	38	48	09	22	02	23	25	00	09	11	15	4	49	12	9	3	13	13.56
2000	01	07	39	53	21	32	42	06	18	14	13	36	14	15	19	12	-22	25	19	49	-20	18	8.87
2000	04	05	39	103	07	05	20	04	18	12	11	16	11	55	0	59	6	18	1	41	5	9	10.51
2000	07	02	37	101	53	02	45	01	19	20	11	25	12	02	6	47	23	0	7	29	21	30	9.83
2000	07	02	38	102	24	02	18	01	19	20	11	22	12	00	6	47	23	0	7	29	21	30	9.83
2000	12	26	41	35	30	30	12	25	17	22	14	45	15	26	18	22	-23	20	19	5	-22	33	9.93
2001	08	19	34	56	48	29	30	19	02	55	14	49	15	23	9	56	12	37	10	30	14	8	8.41
2001	08	19	37	35	30	30	12	19	02	55	16	15	16	52	9	56	12	36	10	34	13	49	9.33
2001	10	17	36	102	24	02	18	16	19	23	10	58	11	34	13	29	-9	22	14	10	-8	37	10.15
2002	09	07	34	56	30	31	06	07	03	10	14	30	15	04	11	4	6	0	11	37	7	52	8.40
2002	11	05	33	56	24	30	00	04	20	34	13	25	13	58	14	42	-15	43	15	24	-17	7	10.17
2002	11	05	35	52	00	30	00	04	20	34	13	42	14	17	14	42	-15	43	15	24	-17	11	10.17
2002	11	05	36	35	36	31	42	04	20	34	14	45	15	21	14	42	-15	44	15	27	-17	24	10.91
2003	01	03	44	03	42	32	30	02	20	23	15	52	16	36	18	56	-22	49	19	44	-25	2	11.19
2003	04	02	38	35	30	30	12	01	19	19	15	57	16	35	0	46	4	56	1	26	5	38	9.98
2003	04	30	26	51	24	35	36	01	12	15	01	44	01	18	2	28	14	35	1	32	6	18	16.06
2003	04	30	29	51	42	32	36	01	12	15	01	48	01	19	2	28	14	35	1	32	6	18	16.06
2003	08	28	41	102	54	05	18	27	17	26	11	17	11	58	10	26	9	47	11	11	10	24	11.09
2003	09	27	38	-111	48	41	48	26	03	09	01	18	01	56	12	13	-1	24	13	5	-3	41	13.19
2003	10	26	41	59	12	32	56	25	12	50	13	19	14	00	14	2	-12	24	14	59	-16	42	14.44
2003	11	25	37	-112	00	41	30	23	22	59	00	02	00	39	16	1	-20	37	17	4	-25	13	15.21
2004	01	22	35	52	00	30	00	21	21	05	14	00	14	35	20	16	-19	45	21	2	-22	7	11.00
2004	05	18	39	51	42	32	36	19	04	52	01	33	00	54	3	40	19	35	2	49	16	34	12.49
2004	09	15	37	59	00	36	36	14	14	29	14	11	14	48	11	34	2	46	12	23	0	4	12.54
2004	09	15	38	50	06	33	18	14	14	29	14	46	15	24	11	34	2	46	12	24	0	-4	12.81
2004	10	15	29	-97	52	30	24	14	02	48	00	01	00	30	13	21	-8	33	14	5	-12	48	11.61
2004	10	15	29	-110	56	32	26	14	02	48	00	52	01	21	13	21	-8	33	14	7	-13	1	12.14
2004	11	13	39	114	48	04	54	12	14	27	10	02	10	41	15	16	-18	6	16	0	-23	19	11.53
2005	02	09	37	51	42	32	36	08	22	28	14	13	14	50	21	33	-14	32	22	15	-15	13	10.17
2005	03	09	27	51	42	32	36	10	09	10	02	51	02	24	23	18	-4	32	22	19	-14	51	17.81
2005	05	09	42	-104	01	30	41	08	08	45	01	39	02	21	3	4	17	20	3	36	22	22	9.05
2005	09	03	36	46	34	33	23	03	18	45	02	29	01	53	10	48	7	35	10	25	13	33	8.22
2005	09	05	41	-110	56	32	26	03	18	45	01	44	02	25	10	56	6	51	11	51	2	34	14.35
2005	10	04	31	52	32	29	37	03	10	28	14	12	14	43	12	42	-4	30	13	31	-10	36	13.58
2005	12	02	28	51	42	32	36	01	15	01	13	27	13	55	16	35	-22	0	17	27	-28	1	13.22
2005	12	02	27	50	10	33	15	01	15	01	13	32	13	59	16	35	-22	0	17	28	-28	2	13.43
2006	02	28	35	51	44	31	40	28	00	31	14	30	15	05	22	45	-7	54	23	19	-5	59	8.65
2006	10	23	30	62	31	27	15	22	05	14	13	14	13	44	13	51	-11	26	14	45	-20	5	15.59
2006	12	21	38	51	44	31	40	20	14	01	13	34	14	12	17	58	-23	26	18	53	-27	52	13.15
2007	09	12	19	50	10	33	15	11	12	44	14	51	15	10	11	21	4	12	12	3	-2	41	12.55
2008	07	03	38	45	06	37	33	03	02	19	16	24	17	02	6	52	22	54	7	30	24	20	8.82
2008	11	28	37	-82	20	29	39	27	16	55	22	30	23	07	16	21	-21	18	17	17	-27	0	13.98
2009	03	27	30	103	07	05	19	26	16	06	11	17	11	47	0	25	2	42	0	55	11	2	11.17
2009	08	21	28	51	52	32	00	20	10	02	15	10	15	39	10	3	11	58	11	1	2	16	17.34
2010	09	09	21	51	42	32	36	08	10	30	14	48	15	09	11	11	5	15	12	5	-6	3	17.59
2010	12	06	36	51	42	32	36	05	17	36	13	27	14	03	16	51	-22	30	17	36	-24	3	10.45
2011	03	05	37	52	32	29	37	04	20	46	14	31	15	08	23	3	-6	6	23	25	1	41	9.53

(01-03)	year,month,day of SS or SR	(12-13)	hour,min of SS or SR	(20-21)	R.A. of moon in hour,min
(04)	lag from SS to MS or MR to SR	(14-15)	hour,min of MS or MR	(22-23)	decl. of moon in ° ,
(05-08)	longitude ° , latitude ° ,	(16-17)	R.A. of sun in hour,min	(24)	arc of light
(09-11)	day,hour,minute of conjunction	(18-19)	decl. of sun in ° ,	SS =	sunset
				SR =	sunrise
				MS =	moonset
				MR =	moonrise





